Comparison of MVI with sparse norm susceptibility inversion accounting for demagnetization

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1 Motivation

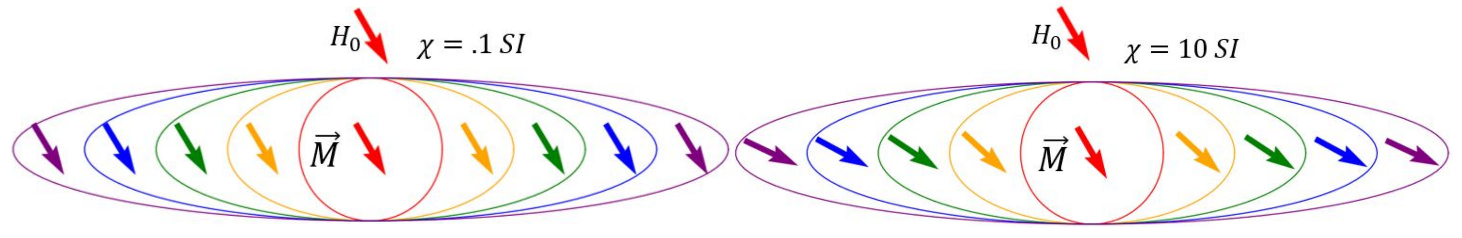
- Magnetic data is linear function of the magnetization $\vec{M}$ of the subsurface
- With no remanence, magnetization is a product of susceptibility and total magnetizing field:

$$\vec{M} = \chi \vec{H}$$
$$\vec{H} = \vec{H}_0 + \vec{H}_S$$

- "Demagnetizing" secondary field $\vec{H}_S$ opposes magnetization internal to body
- At low susceptibility, induced magnetization due to secondary field can be ignored, magnetization can be approximated as:

$$\vec{M} \approx \chi \vec{H}_0$$

- At higher susceptibilities, demagnetization is function of shape and susceptibility of body.
- If not equidimensional, demagnetization rotates the magnetization and changes the characteristics of TMI anomaly

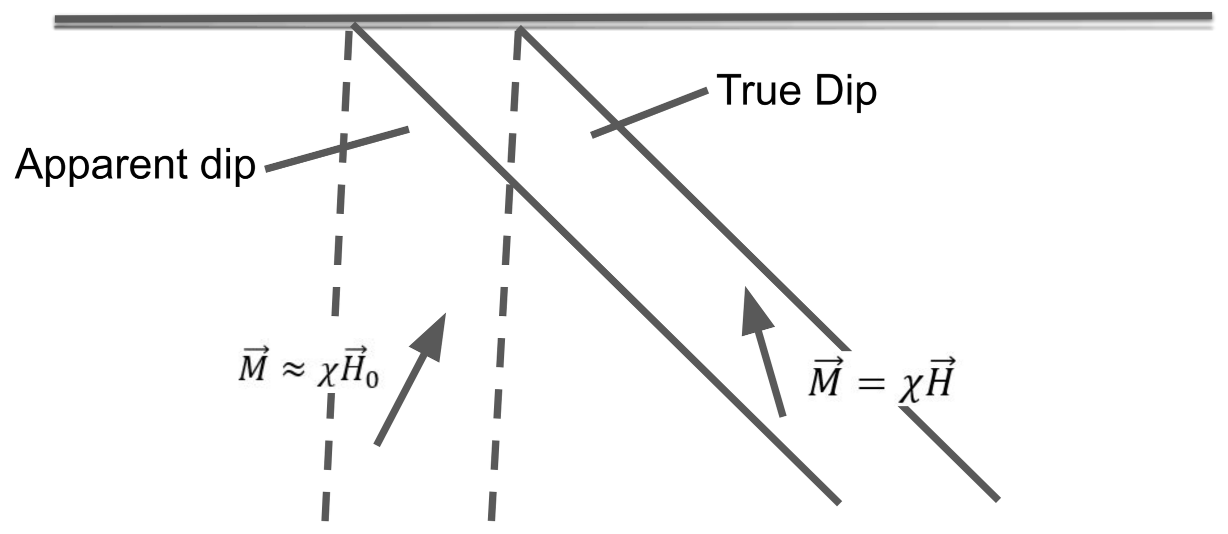


- Two main approaches to forward model and invert data affected by self-demagnetization
  - Modeling in terms of high susceptibility
  - Modeling in terms of total resultant magnetization (magnetic vector inversion)

2 Objectives

1: Forward model data from synthetic model inspired by the Osborne deposit, where not accounting for demagnetization led to incorrect interpretation of dip

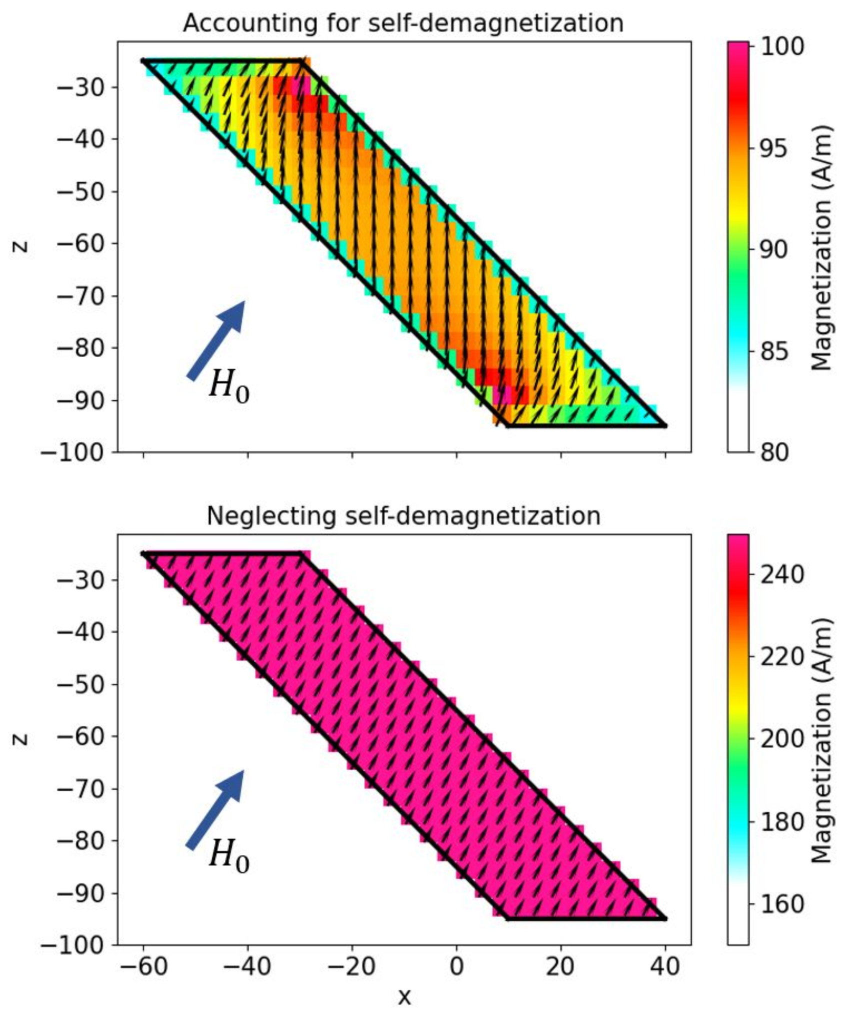
2: Compare different inversion methods (MVI, low susceptibility, high susceptibility) using different inversion approaches (smooth, sparse norm)

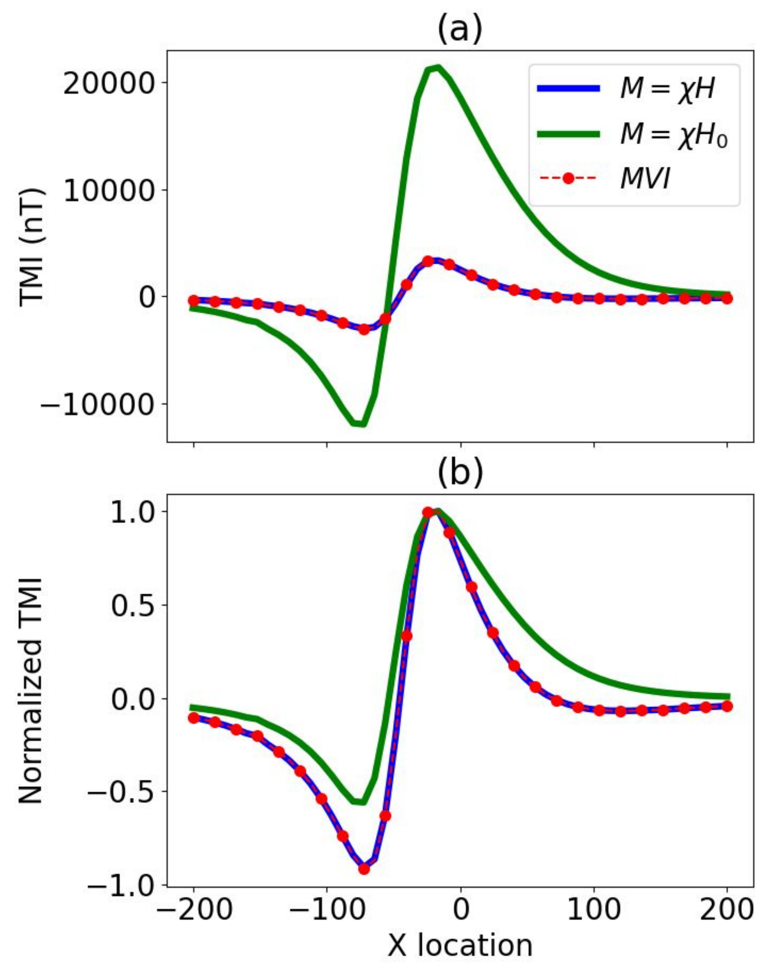


Adapted from Clark, 2000

3 Forward Modeling

- A grid of TMI data forward modelled for a plate with χ=6 SI extending 200m in both y directions
- Both direction and amplitude of magnetization are significantly altered
- Both amplitude (a) and shape (b) of corresponding TMI anomaly are altered
- Using the computed magnetization and forward modeling with MVI indicates the capability of modeling demagnetization





4 Smooth Inversion

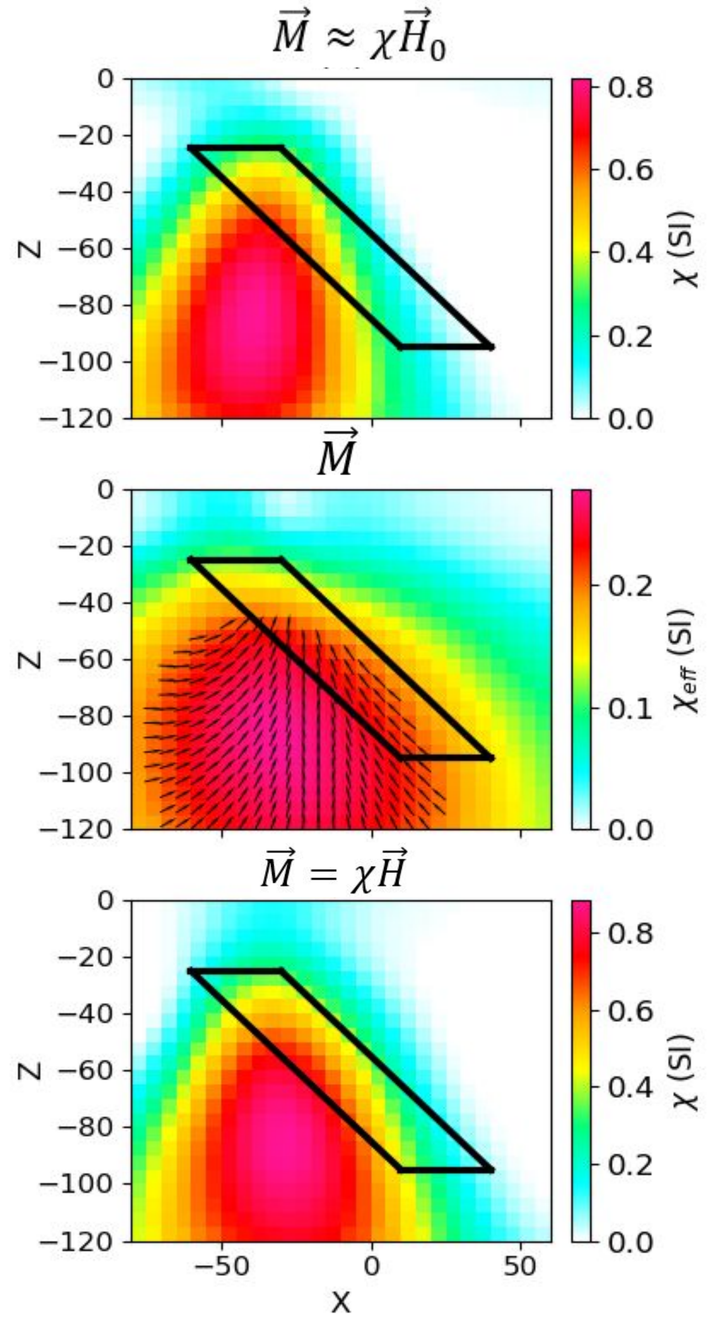
- Objective Function contains a data misfit term and regularization term:

$$\phi = \phi_d + \beta \phi_m$$

- Regularization Term:

$$\phi_m = \alpha_s \int w_s |m - m_{ref}|^p dv + \sum_{j=x,y,z} \alpha_j \int w_j \left|\frac{\partial m}{\partial i}\right|^q dv$$

- Standard smooth inversion is choice of p=2 and q=2
  - Tends to overly smooth model, recover lower physical property values
- Sensitivity weighting helps to place model at depth and compact the model
- We invert the data using standard susceptibility inversion, MVI, and high susceptibility inversion
- At recovered susceptibilities, self demagnetization effects are not adequately simulated
- High susceptibility inversion improves only slightly as compared to low susceptibility
- MVI is able to fit data with lower effective susceptibility and a smoother model as direction of magnetization not constrained



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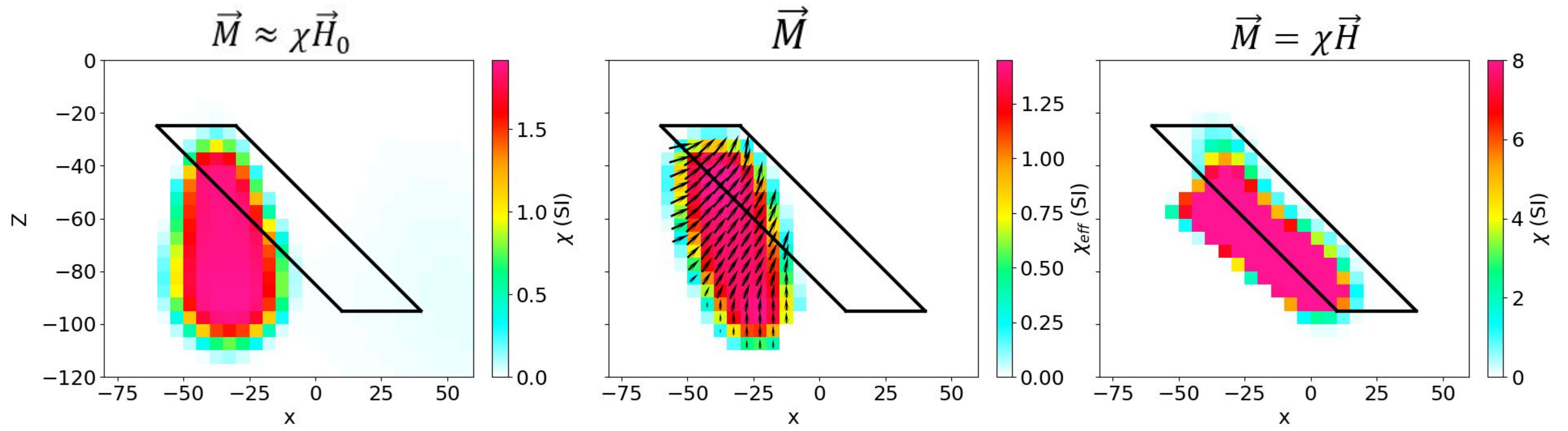
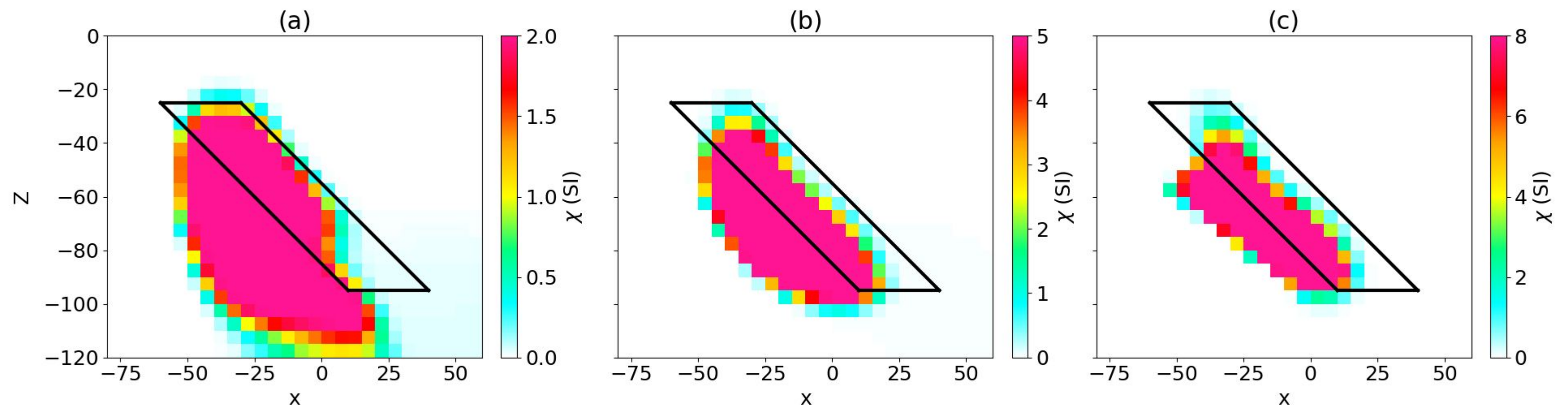
5 Sparse Inversions

- Iteratively reweighted sparse norm algorithm allows for choice of p or q between 0 and 2
- Lower norm choices recover more compact (p) and sharper (q) models
- Norms of 1/2 are chosen for both p and q are chosen for all standard susceptibility (a), MVI (b), and high susceptibility (c)
- The low susceptibility inversion recovers a vertical dip in the center of the plate, consistent with the initial modeling at Osbonre
- While MVI model slightly improves the dip in the center of the plate, shows more consistency with linear code
- The sparse high susceptibility inversion gives the best indication of the dip of the plate



- Sparse norms can over-compact models, especially in combination with sensitivity weighting in high susceptibility inversion
- Bound constraints to avoid over compacting the model
- Bound constraints of 2 (a), 5 (b) and 8 (c) SI are shown
- All inversions improve dip and location, but bound constraints near to the true susceptibility (6 SI) do a better job



6 Conclusion

- Magnetic vector inversion can model demagnetization, but increased number of model parameters adds additional non-uniqueness to space of models that can fit the data
- Sparse-norm high susceptibility inversion can improve on recovered models if prior information is available
- Amplitude and direction of magnetization are a function of body geometry if susceptibility is high, it is not as easy to include prior information in MVI as it is in susceptibility inversion

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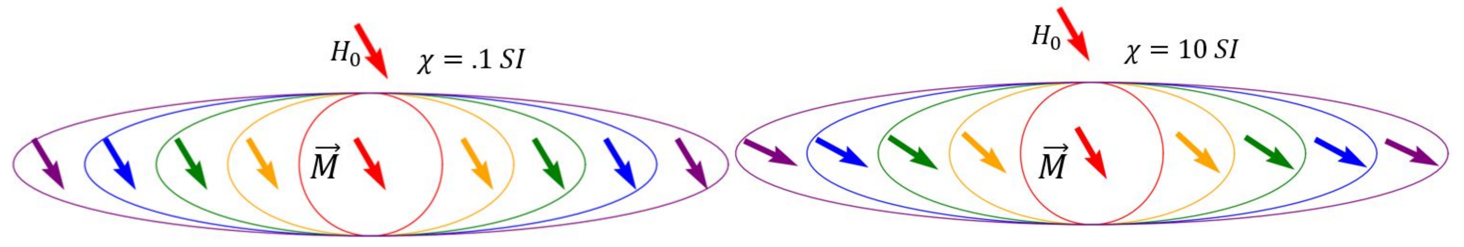
1 Motivation

- At Osborne deposit, not accounting for demagnetization effect led to incorrect interpretation of dip and misled drilling
- Magnetic data is linear function of magnetization $\vec{M}$
- With no remanence, $\vec{M} = \chi\vec{H}$
  $\vec{H} = \vec{H}_0 + \vec{H}_S$
- "Demagnetizing" secondary field $\vec{H}_S$ opposes internal magnetization
- At low susceptibility: $\vec{M} \approx \chi\vec{H}_0$
- At higher susceptibilities, demagnetization is function of shape and susceptibility of body.
- Demagnetization can rotate the direction of magnetization
- Two approaches for handling self-demagnetization
  - Modeling in terms of high susceptibility using partial differential equations
  - Modeling in terms of total resultant magnetization (magnetic vector inversion) using integral formulation



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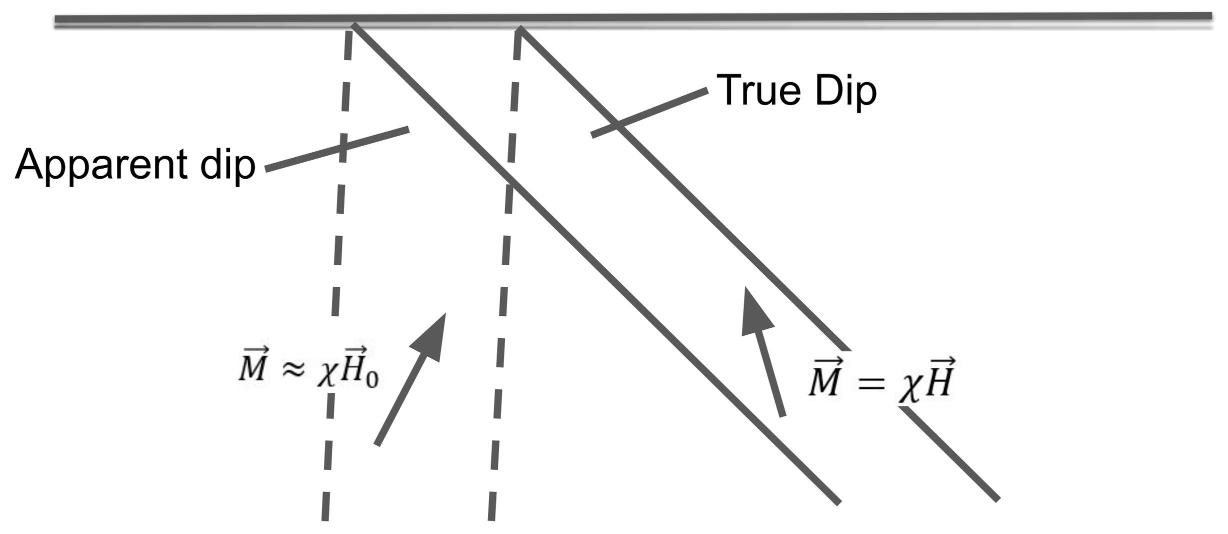
2: Compare different inversion methods (low susceptibility, MVI, high susceptibility) using different inversion approaches (smooth, sparse norm)



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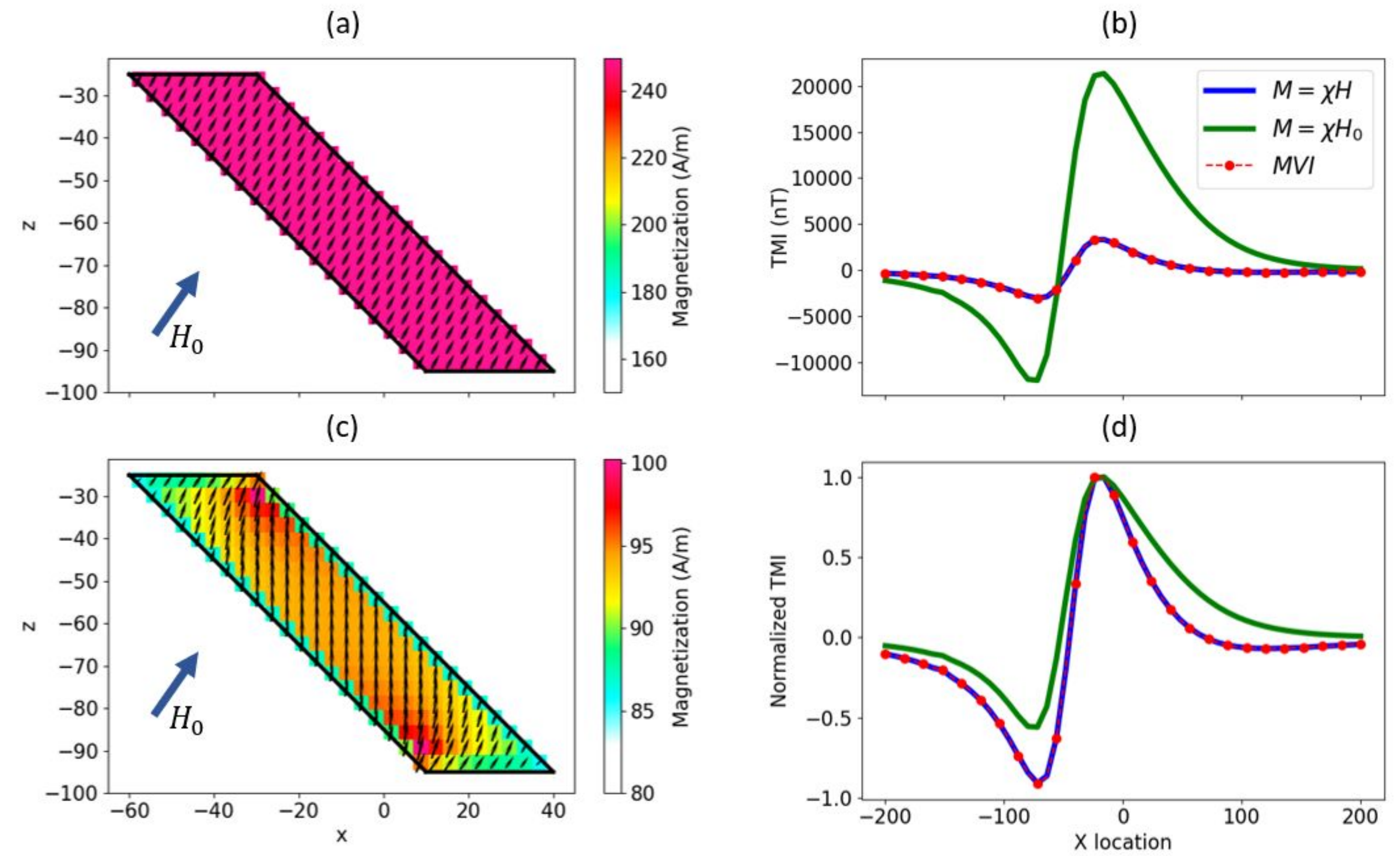
3 Forward Modeling

- Forward modelled for a plate with χ=6 SI extending 200m in both y directions
- Low-susceptibility (a) magnetization in direction of inducing field
- High susceptibility (c) magnetization non uniformly attenuated and rotated
- The TMI anomaly (b) is much stronger in the
- Normalized TMI anomaly (d) is still significantly altered



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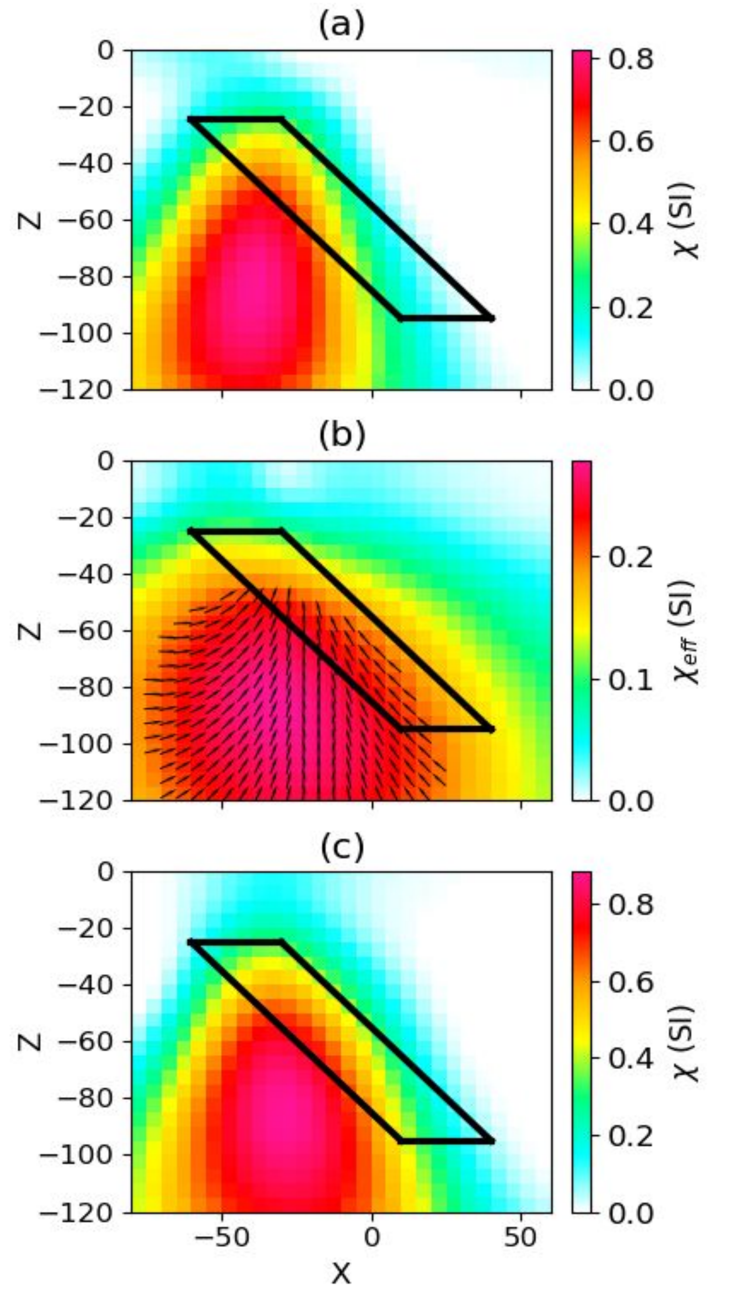
- Standard smooth *L2* inversion is choice of *p*=2 and *q*=2
  - Tends to overly smooth model, recover lower physical property values
- Sensitivity weighting helps to place model at depth
- Low susceptibility inversion (a) indicates dip slightly away from plate
- MVI (b) recovers a larger volume of magnetization, is aligned near the top of the plate
- High susceptibility (c) slightly improves dip and location near the top of the plate
- All three give a poor indication of the extent and dip of the plate



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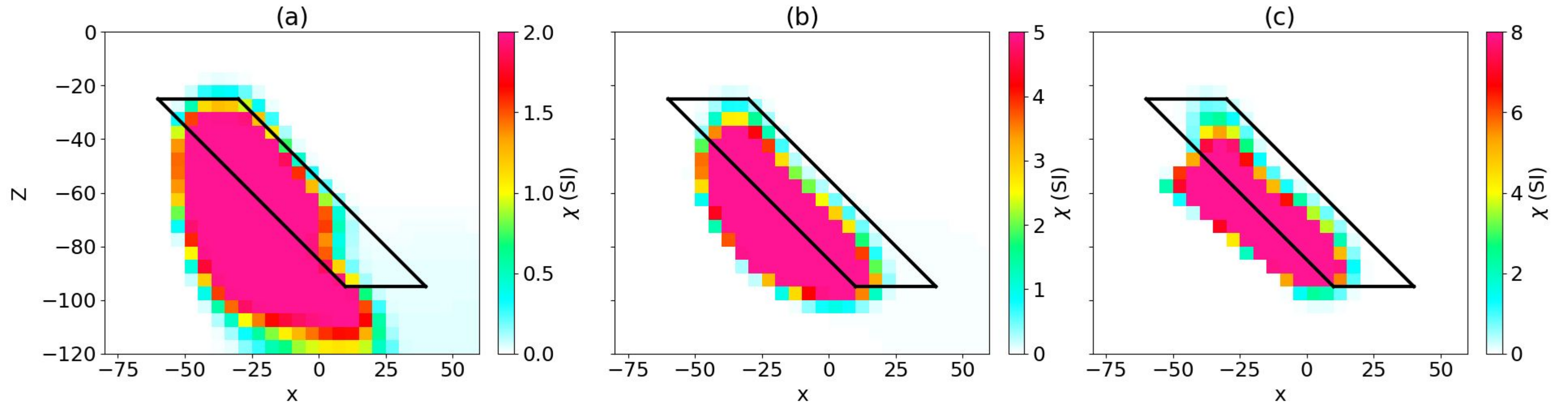

- All inversions improve dip and location, but bound constraints near to the true susceptibility (6 SI) do a better job
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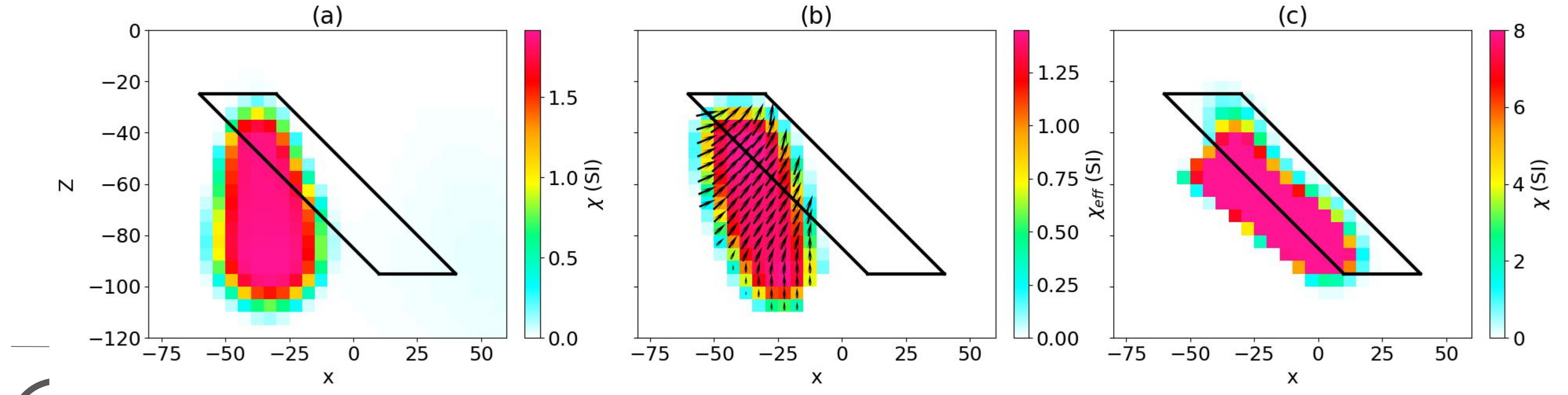

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- Amplitude and direction of magnetization are a function of body geometry if susceptibility is high
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