Parametric Inversions using Radial Basis Functions

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IMAGE '25

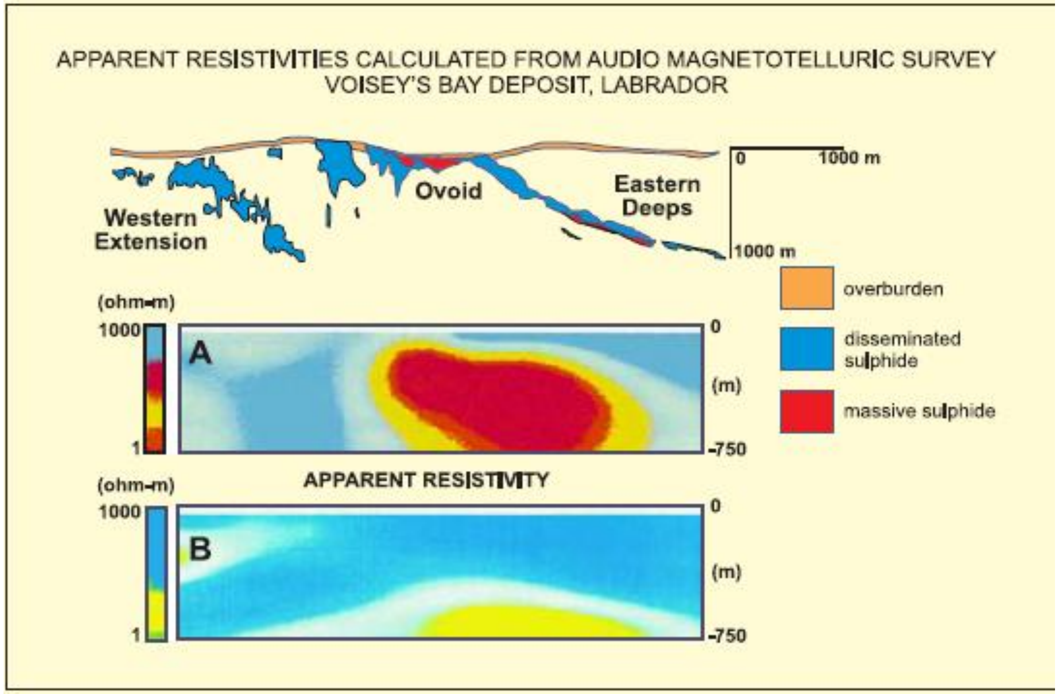
Modeling and Drilling for Mineral Exploration

Outline

- Motivation
- Implementation
- Results
- Future work

Motivation

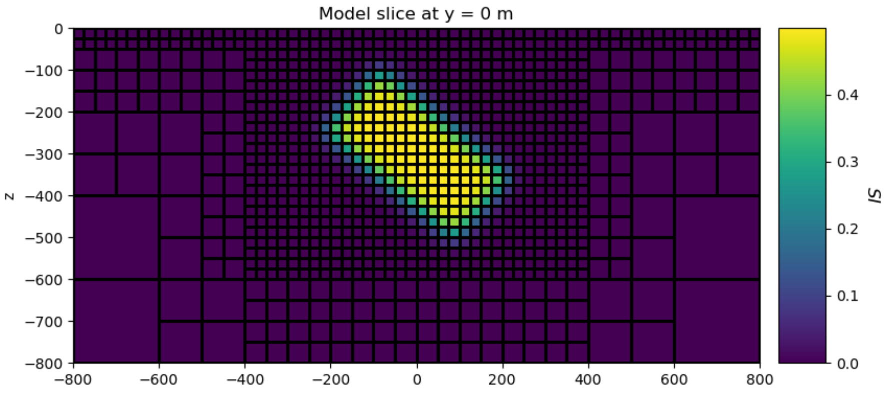
- Geological targets present large physical property contrasts
- But are rarely simple shapes – curvy, irregular, non-smooth
- Traditional inversions produced smoothing effect.

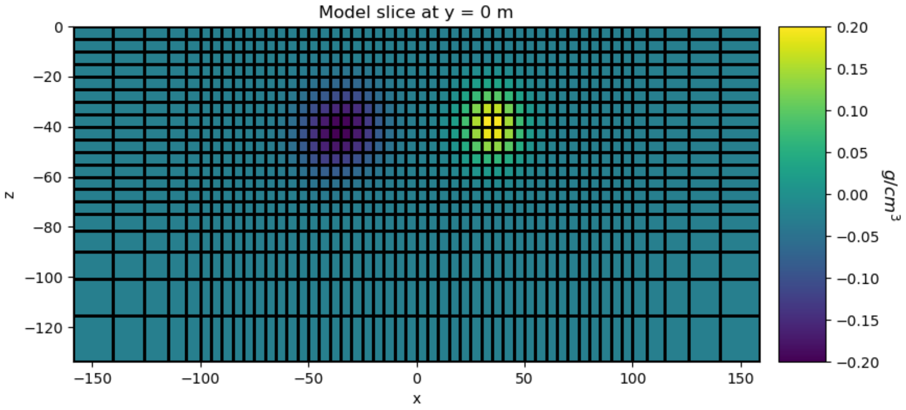


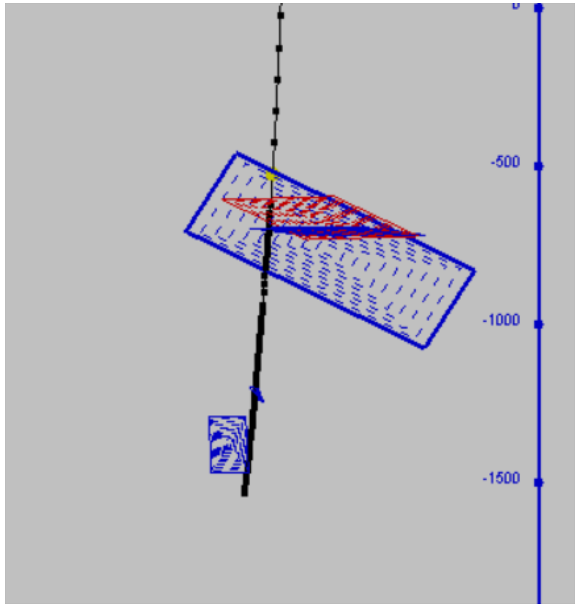
King 2007

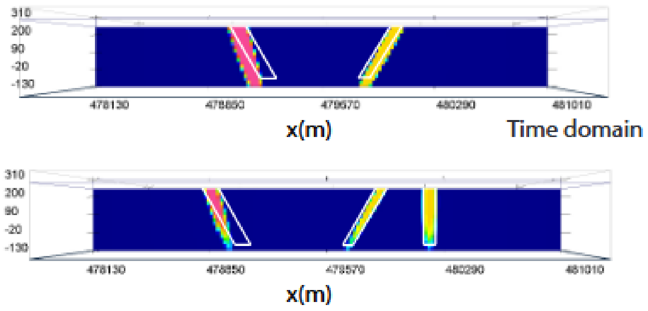
Motivation

- Large contrasts can be recovered using parametric models
- Parametric ellipses/prisms reduce parameters, but cannot capture complex shapes





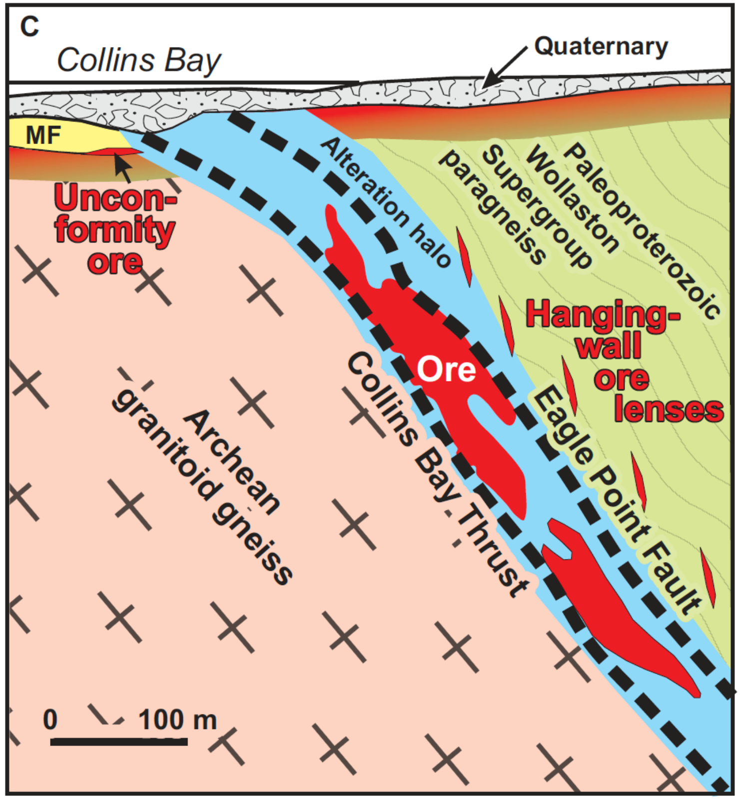


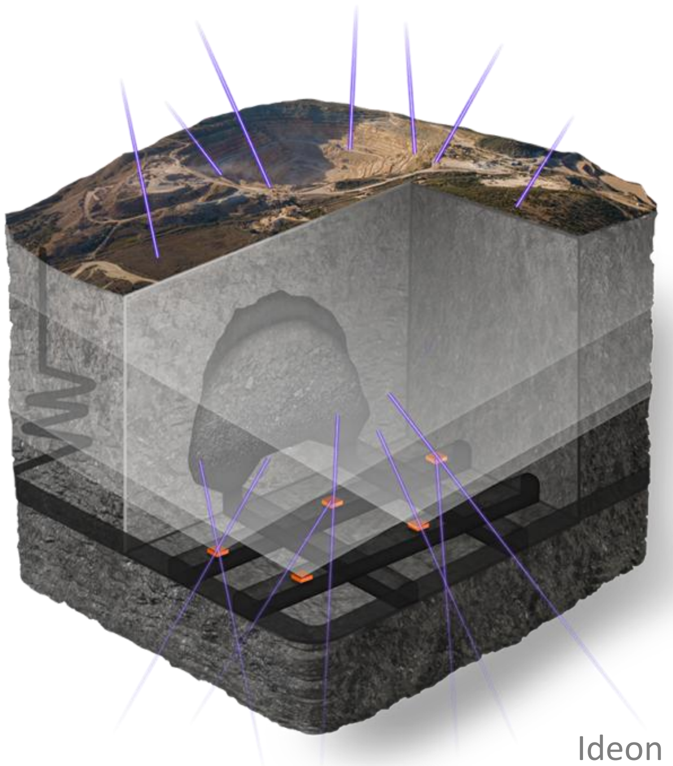


McMillian 2017

Goal

- What if target geology cannot be represented by prism or ellipses?
  - Or number of shapes required to represent target is unknown?
- Use RBFs to invert for arbitrary shapes without needing to predefine geometry or number of bodies

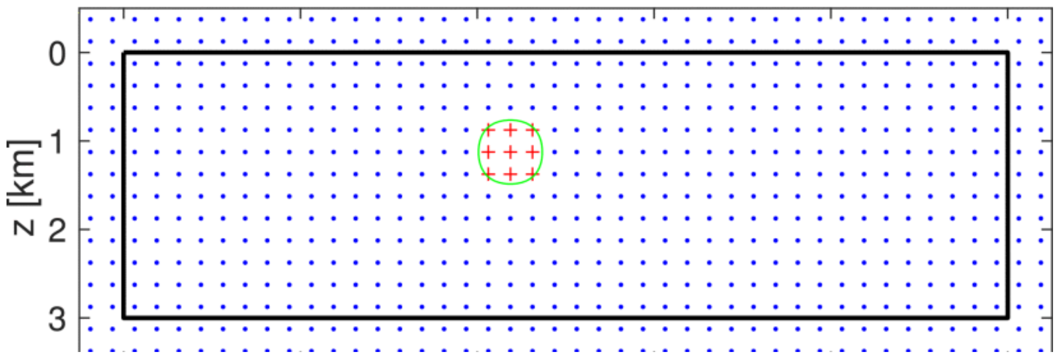


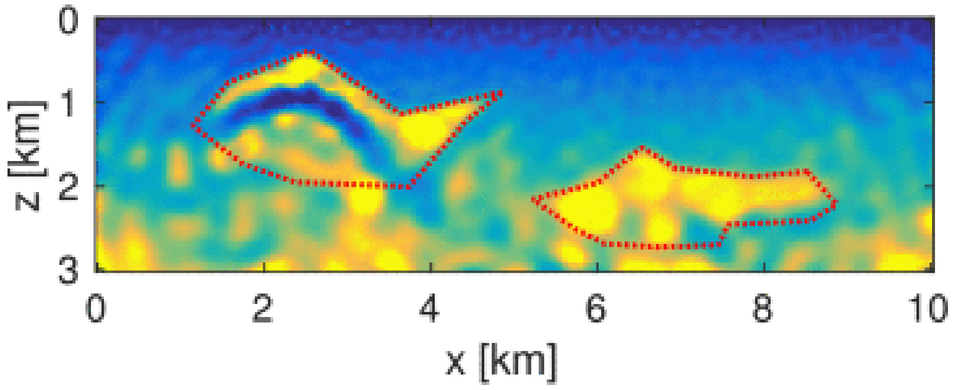


Radial Basis Functions

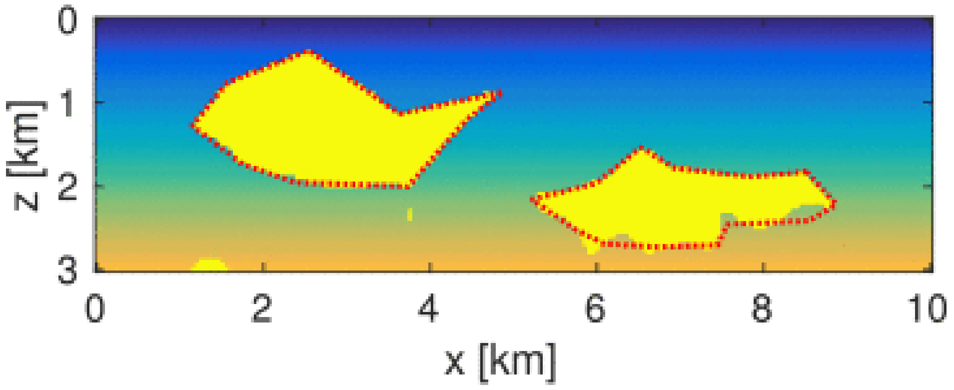
- Use a set of Radial Basis Functions to parameterize instead
- Motivated from Kadu et al (2017) work in Seismic FWI, and Aghasi et al (2011).

RBF mesh





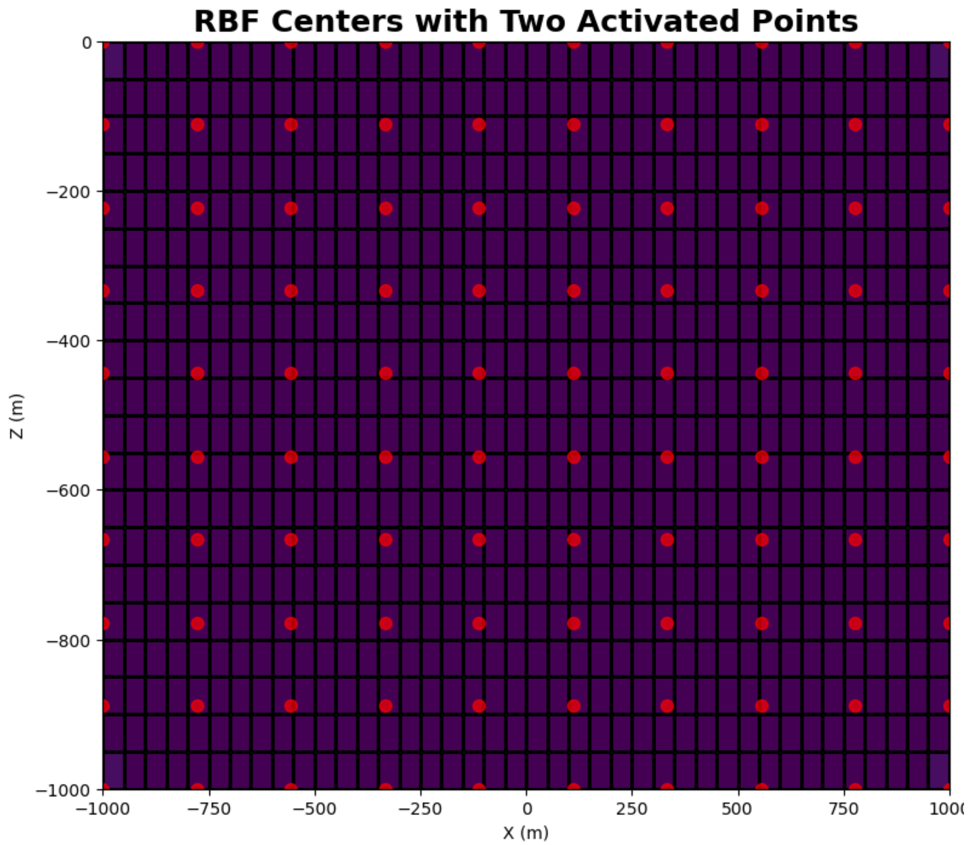
Conventional FWI

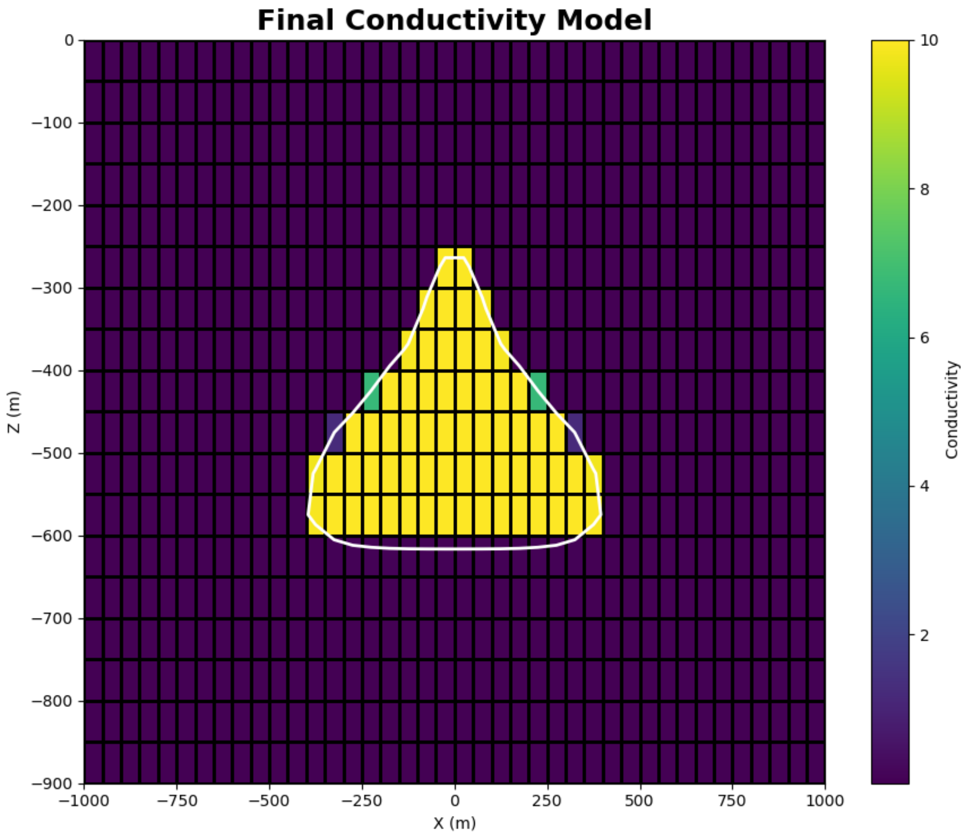


Level set FWI

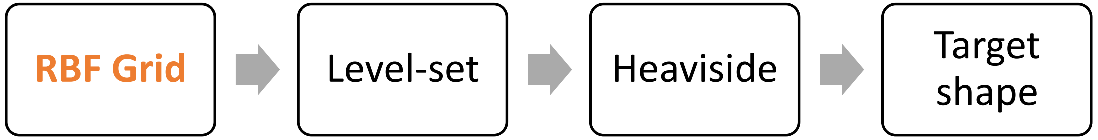
Parameterization

Define RBF Grid → Level-set Function → Heaviside Transform → Create Target shape





Parameterization



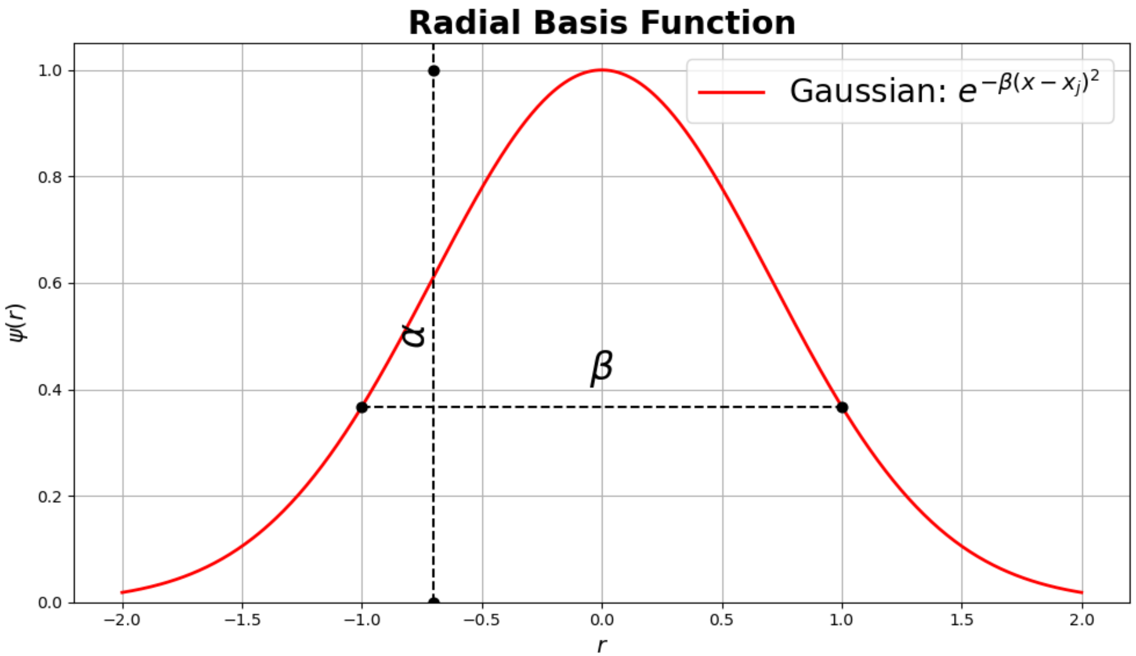
- Define RBFs (Ψ) on a sub-grid
- Choose possible parameters for each RBF:

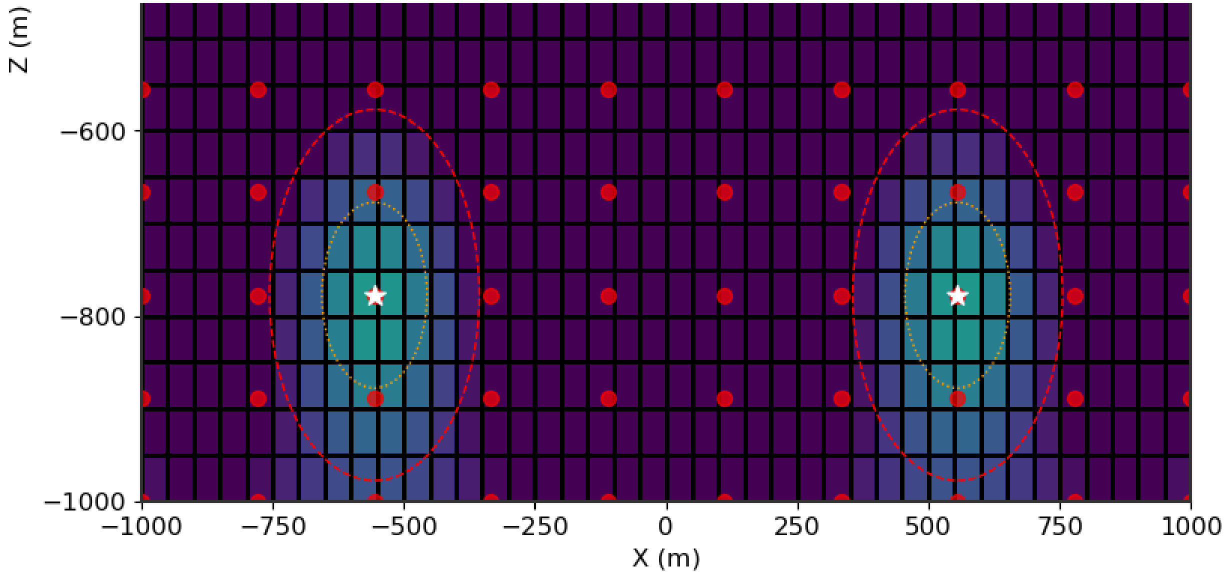
$$\Psi = \alpha\, e^{-\beta\left(x_{mesh} - x_{rbf}\right)^2}$$

Expansion coefficients

Dilation coefficients

RBF Centre Coords





Parameterization

RBF Grid → **Level-set** → Heaviside → Target shape

- Any smooth level-set function (φ) can be recreated by a linear combination of a set of Radial Basis functions (ψ).

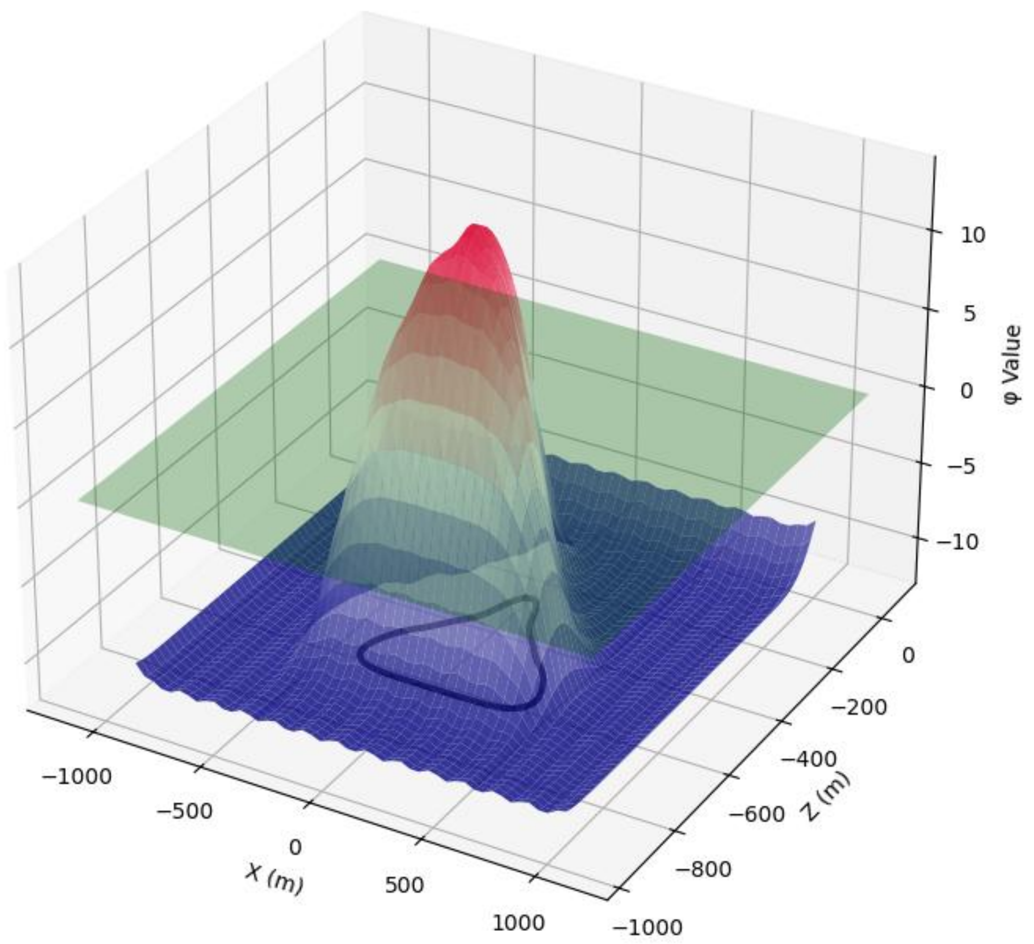
- RBFs define Level-set function :

$$\varphi(x, \alpha) = \sum_{j=1}^{n\_rbf} \alpha_j \psi(\| \beta(x - x_j) \|)$$

$$\psi(r) = e^{-\frac{r^2}{2\sigma^2}}$$

- Use level-set to define edge of target ($\varphi = 0$)
  - If $\varphi > 0$: Inside | $\varphi < 0$: Outside

Level Set Function (φ) with Zero Level Contour

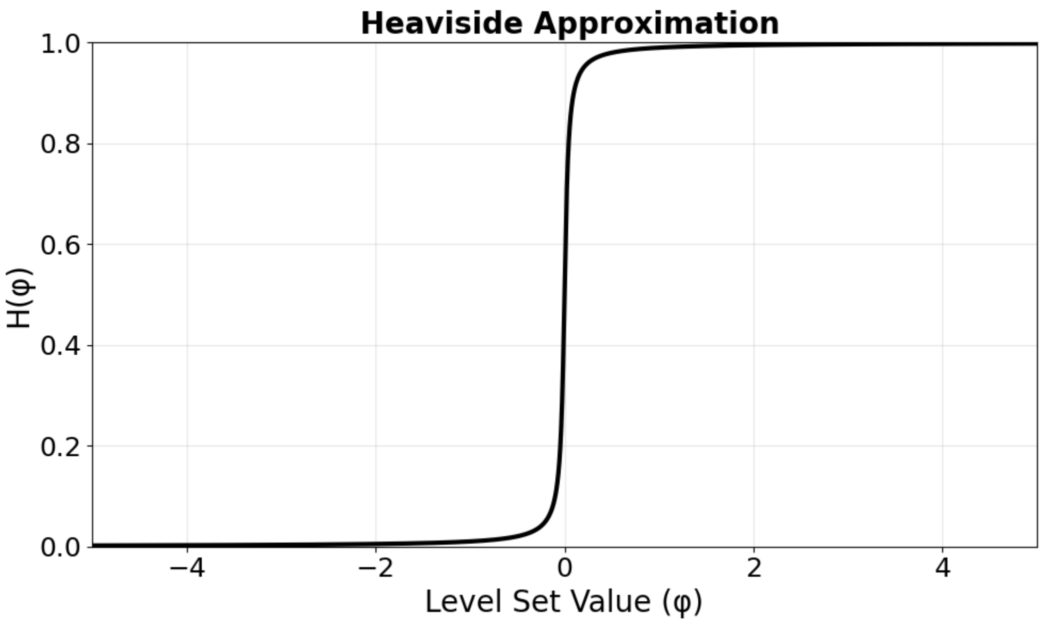


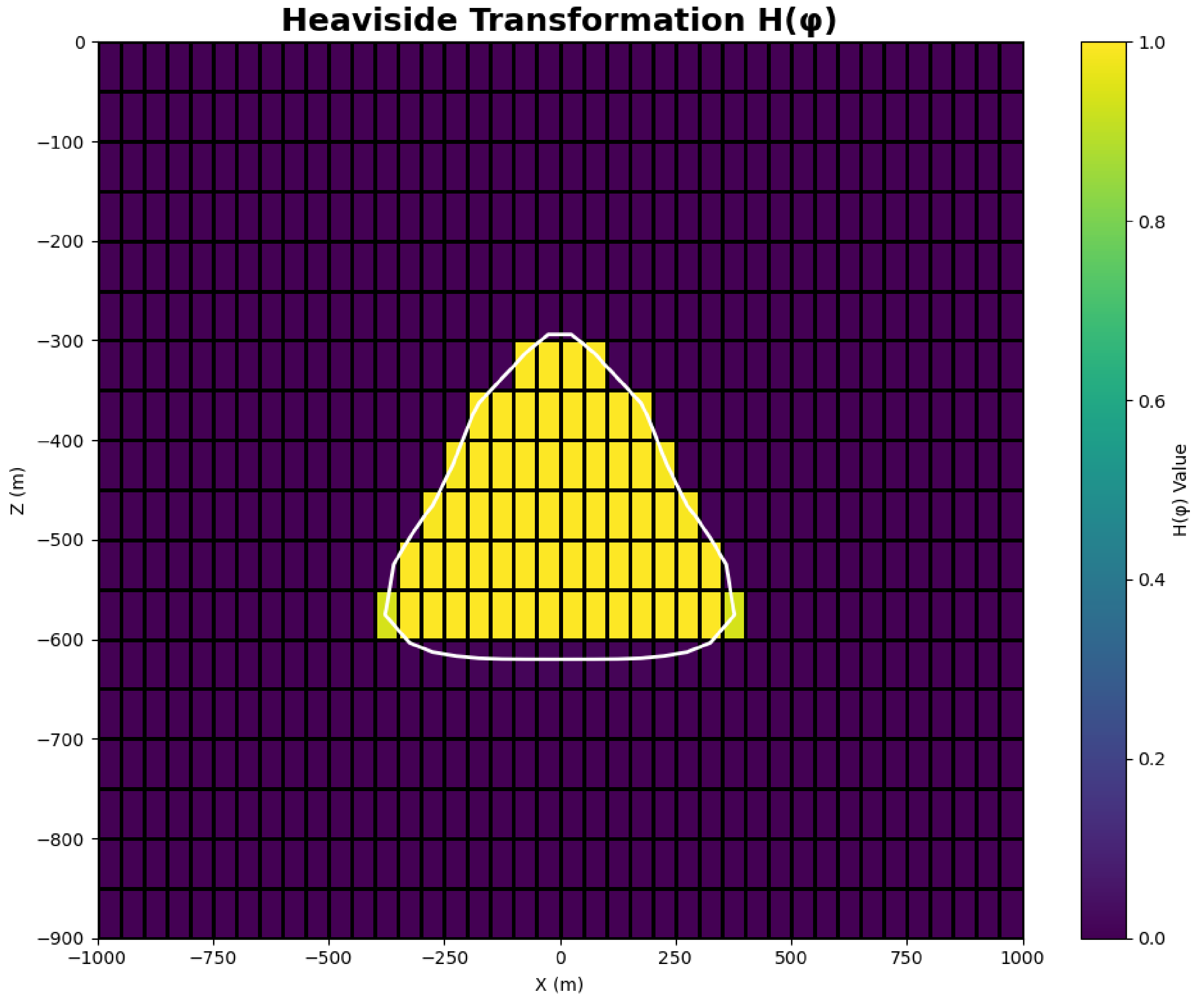
Parameterization

RBF Grid → Level-set → **Heaviside** → Target shape

- Binarize Level set function with a sigmoid or Heaviside function:

$$H_\epsilon(\varphi) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{\pi\phi}{\epsilon}\right)\right)$$





Parameterization

RBF Grid → Level-set → Heaviside → Target shape

- For a problem of form

$$F(\boldsymbol{m}) + \eta = d$$

$$\boldsymbol{m}(x,p) = \boldsymbol{m_0}(x) + \boldsymbol{H}(\boldsymbol{\phi}(x,p))\, \boldsymbol{m_p}(x)$$

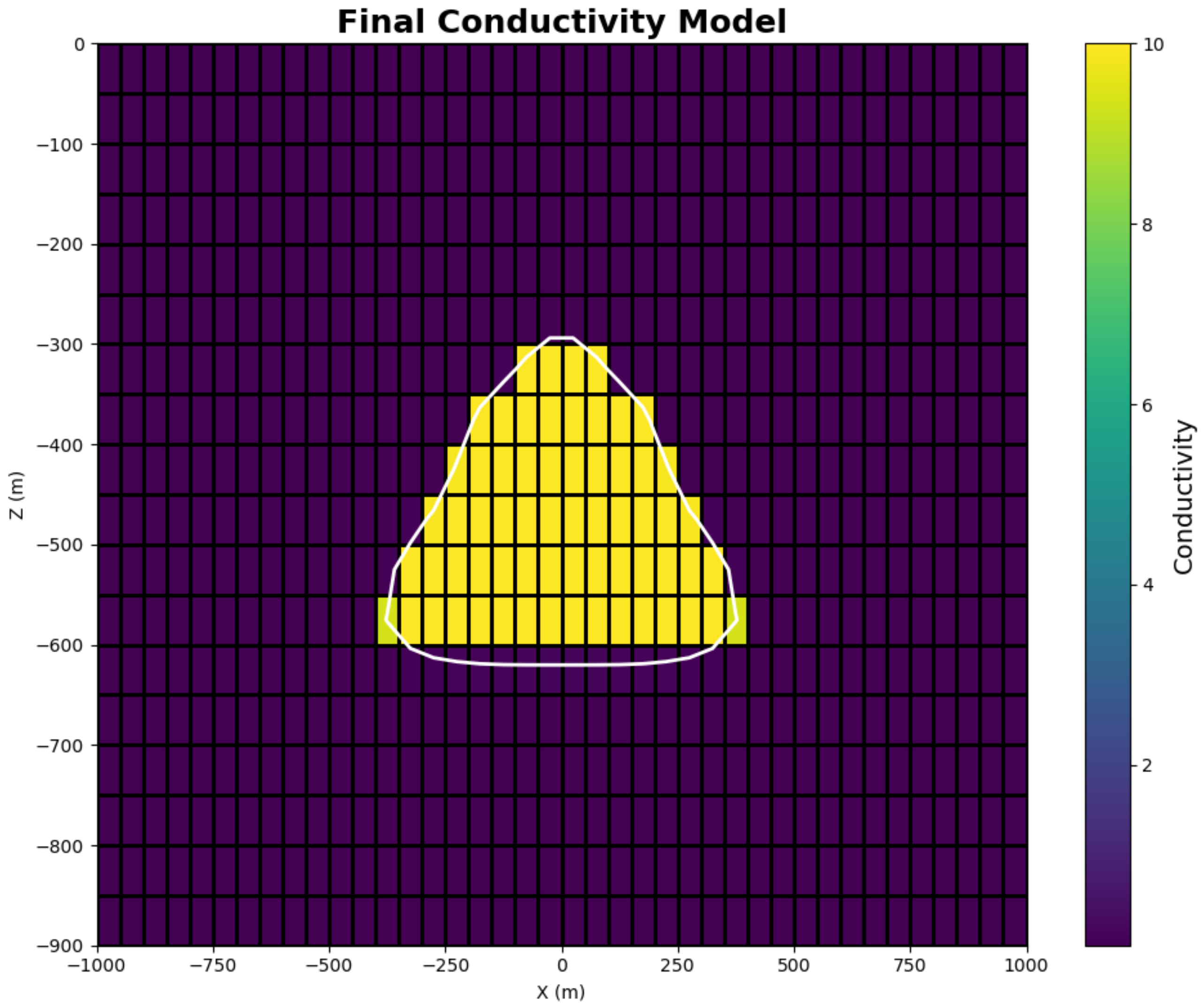
m **=** model space***;*** *d* = data ***;*** η **=** noise

p = Inversion parameters

$\mathbf{m_0}$ = Background physical (***10^{-4}***)

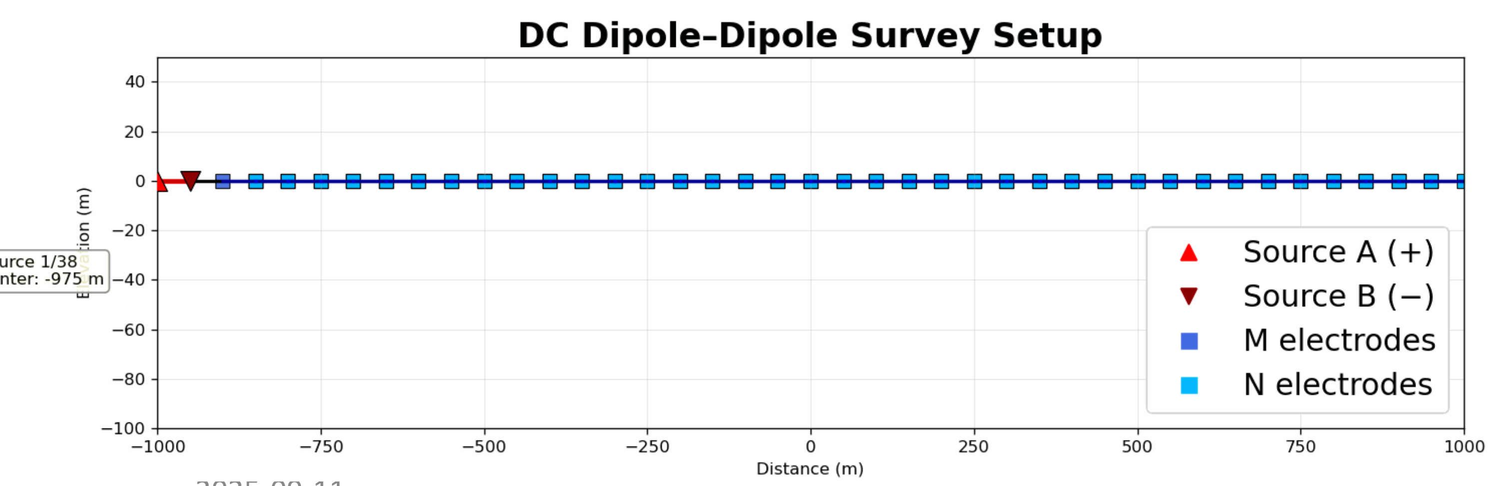
$\mathbf{m_p}$ = Physical property contrast (***10***)

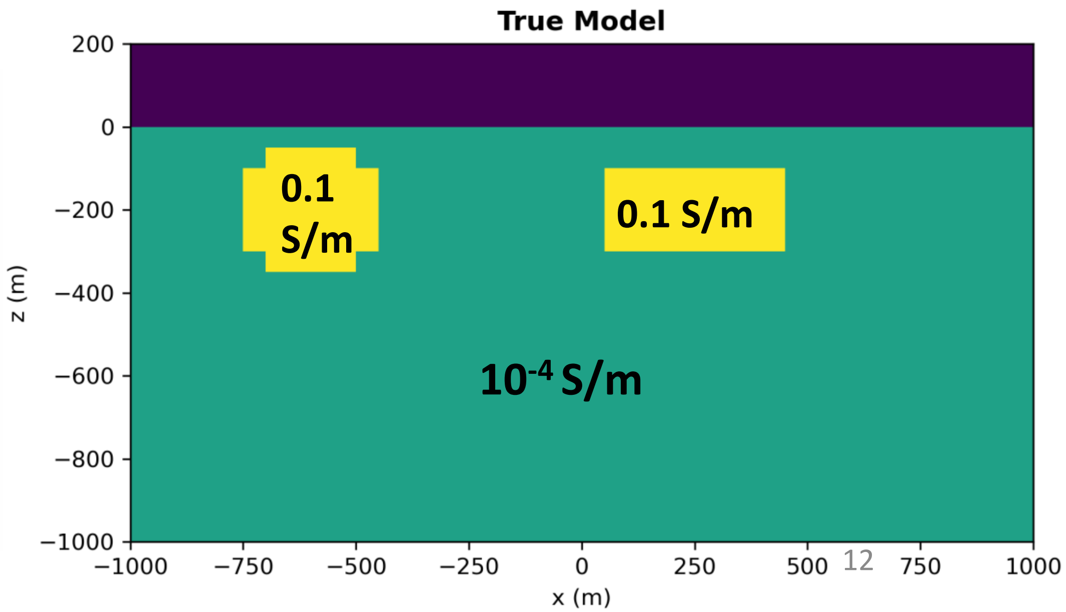
- Target - background



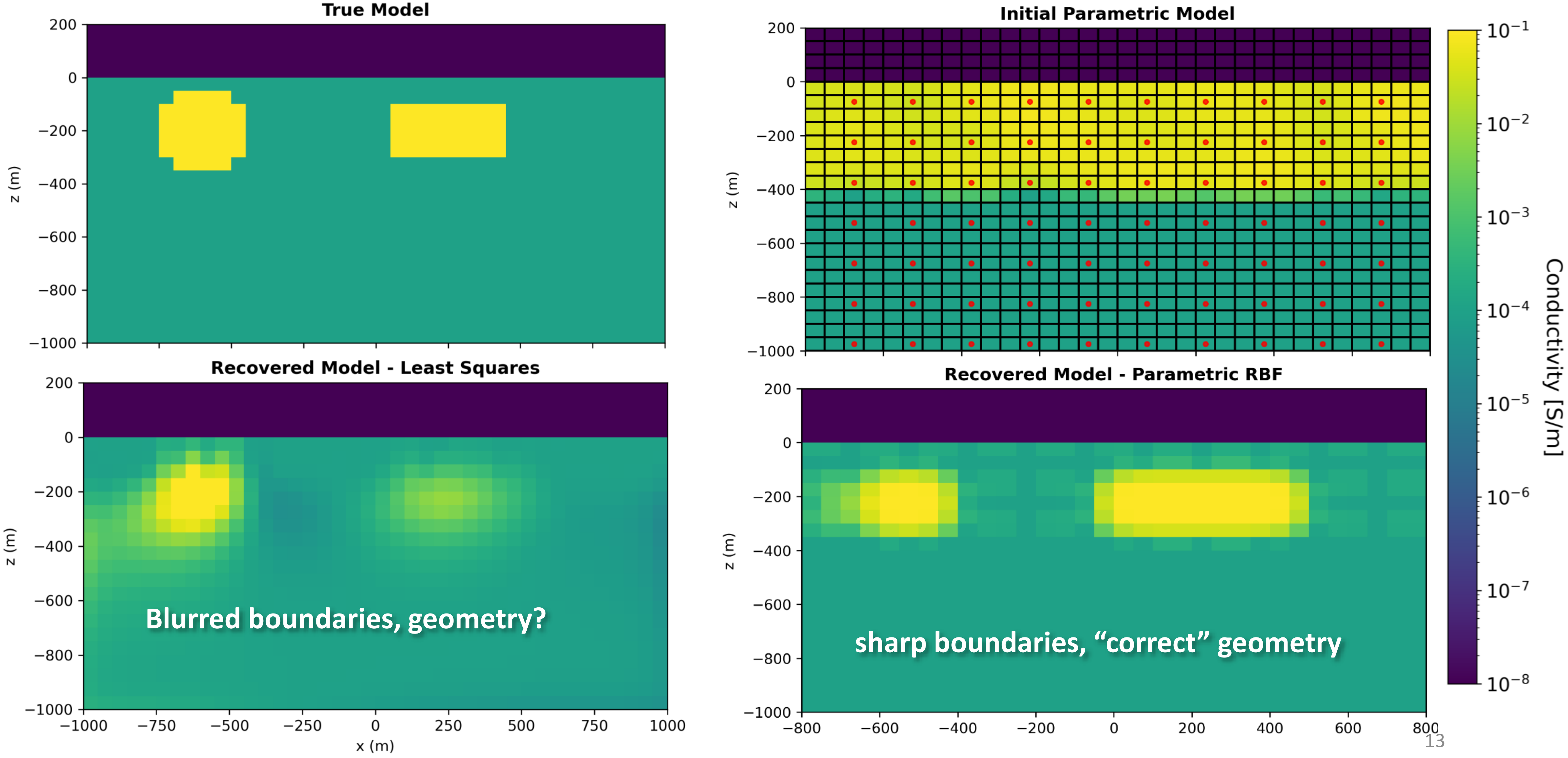
Results – DC-Resistivity Problem

- **2D** test set up as a **DC-Resistivity Dipole-Dipole Array**
- Simulating high conductivity targets
- **Line length**: 4km (-2000m to 2000m)
- **Source electrode (A-B) separation**: 50m
- **Receiver electrode (M-N) separation**: 50m moving outwards from source.
- Inverted using SimPEG **without explicit regularization**.

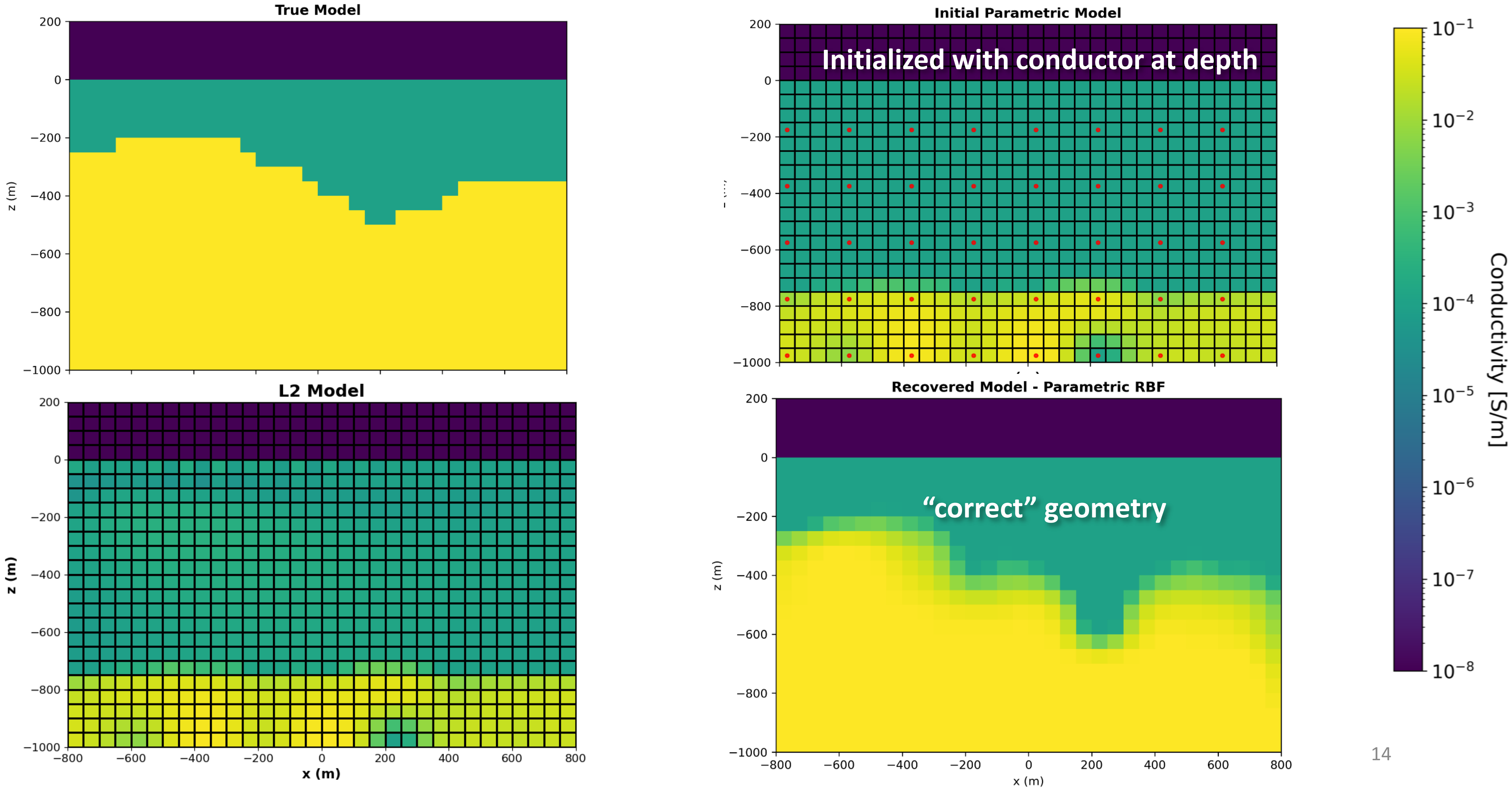




Results – Two bodies (DC-R)



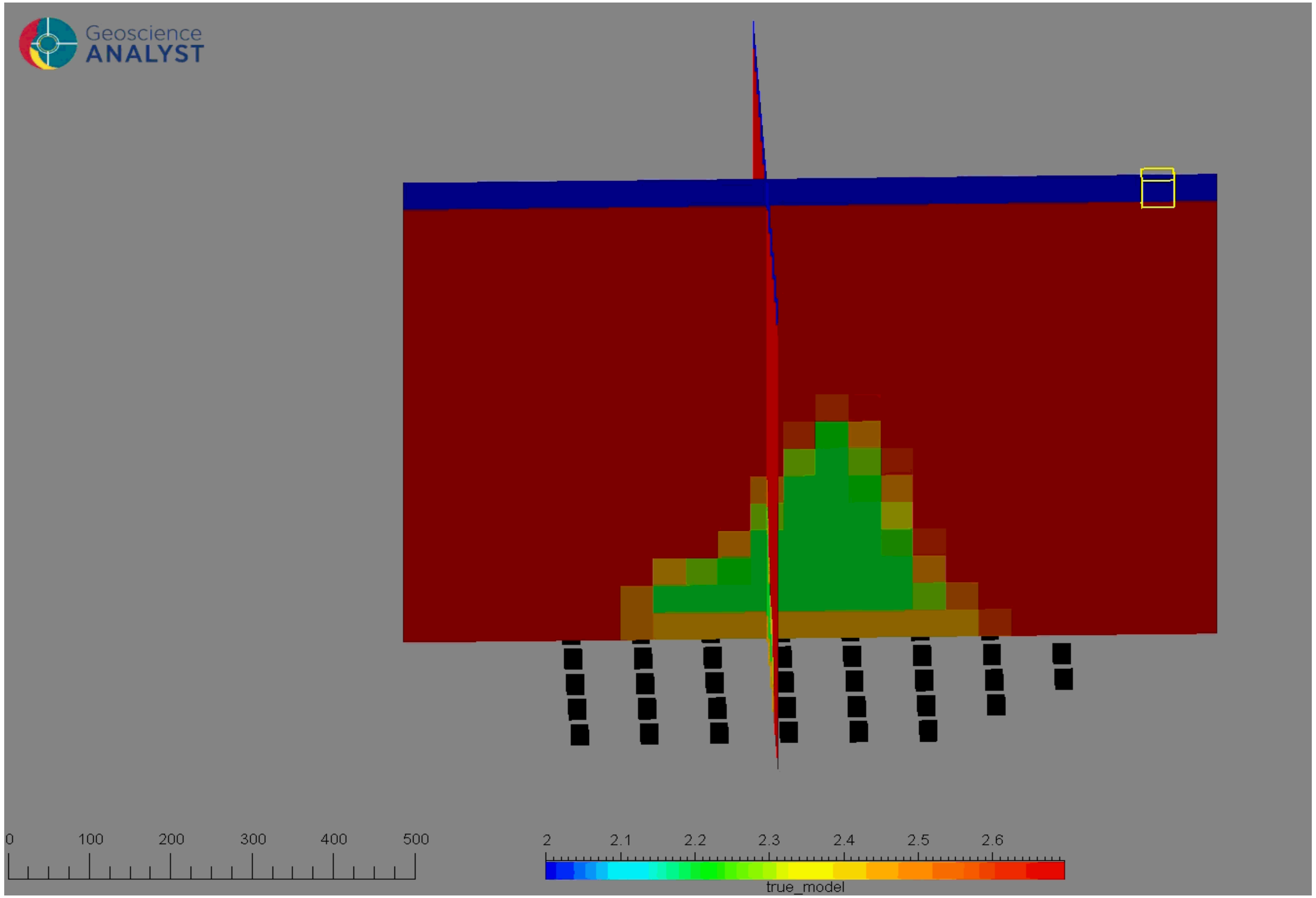
Results – Depth to basement (DC-R)



Results – Muon Tomography

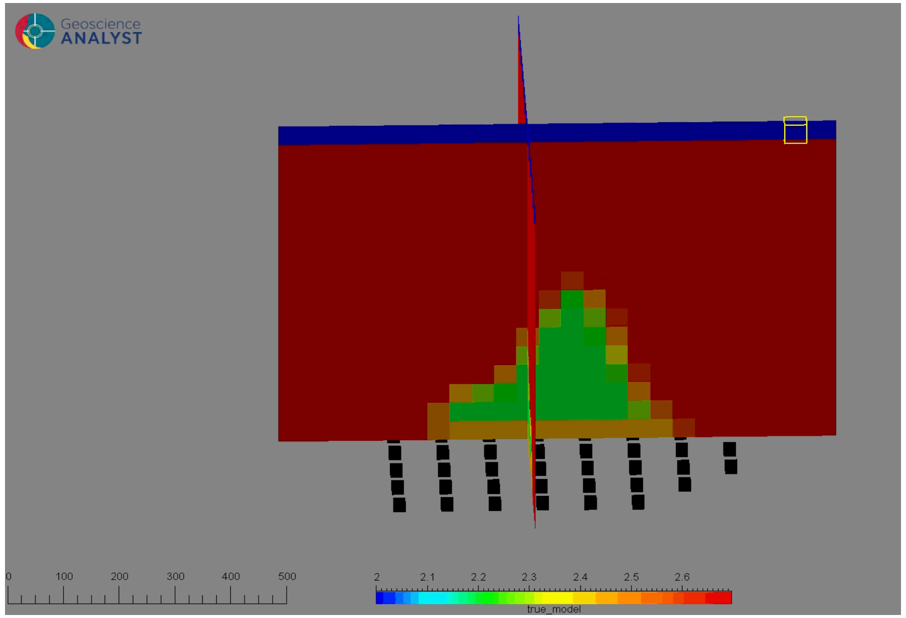
- Testing in synthetic **3D Muon Tomography survey**
- Simulating a low-density body (cave)
- **49 Sensors** below target
- **Data**: Predicted muon counts
- **Exposure time** : 90 days

- Simulation implemented in SimPEG — courtesy of Ideon

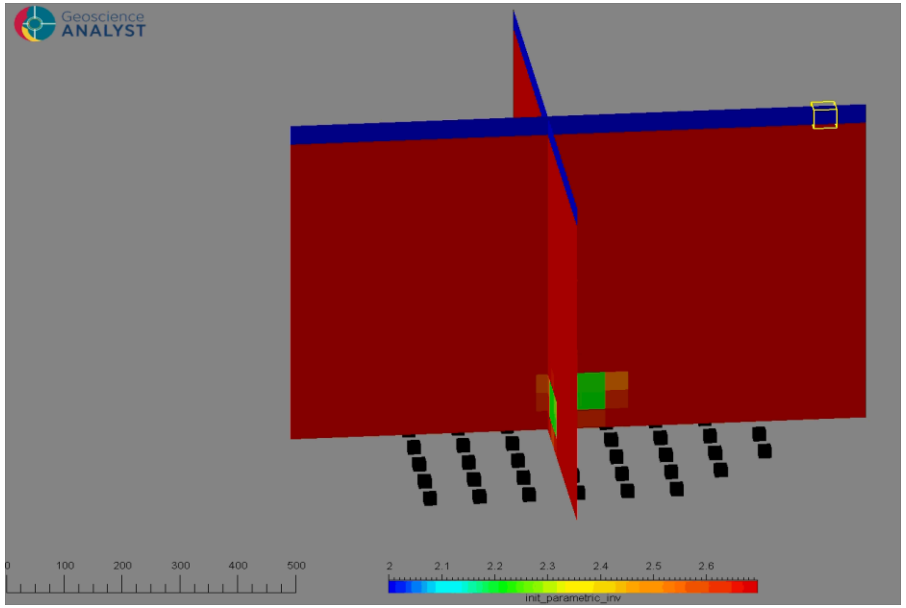


Results – Muon Tomography

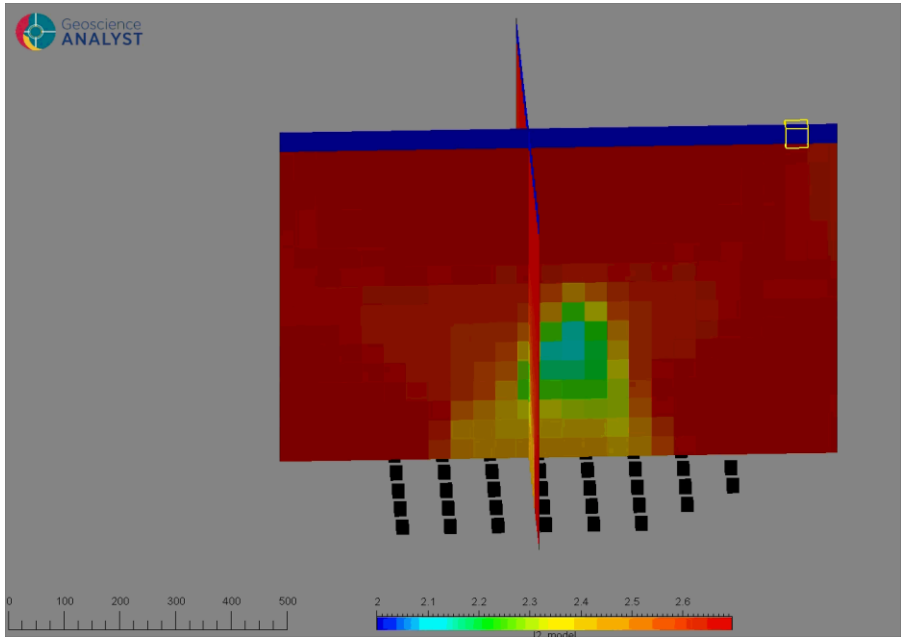
True model



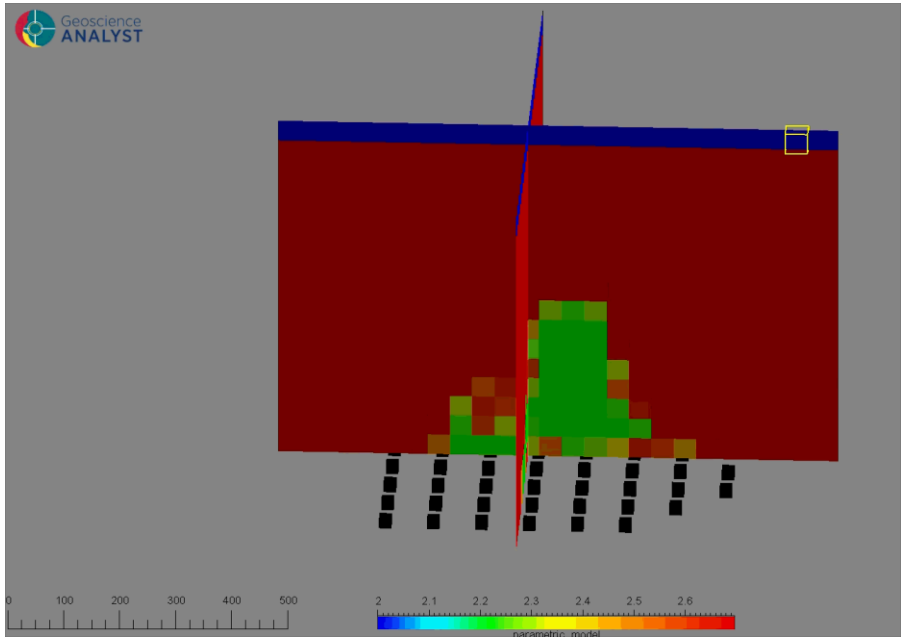
Initial param model



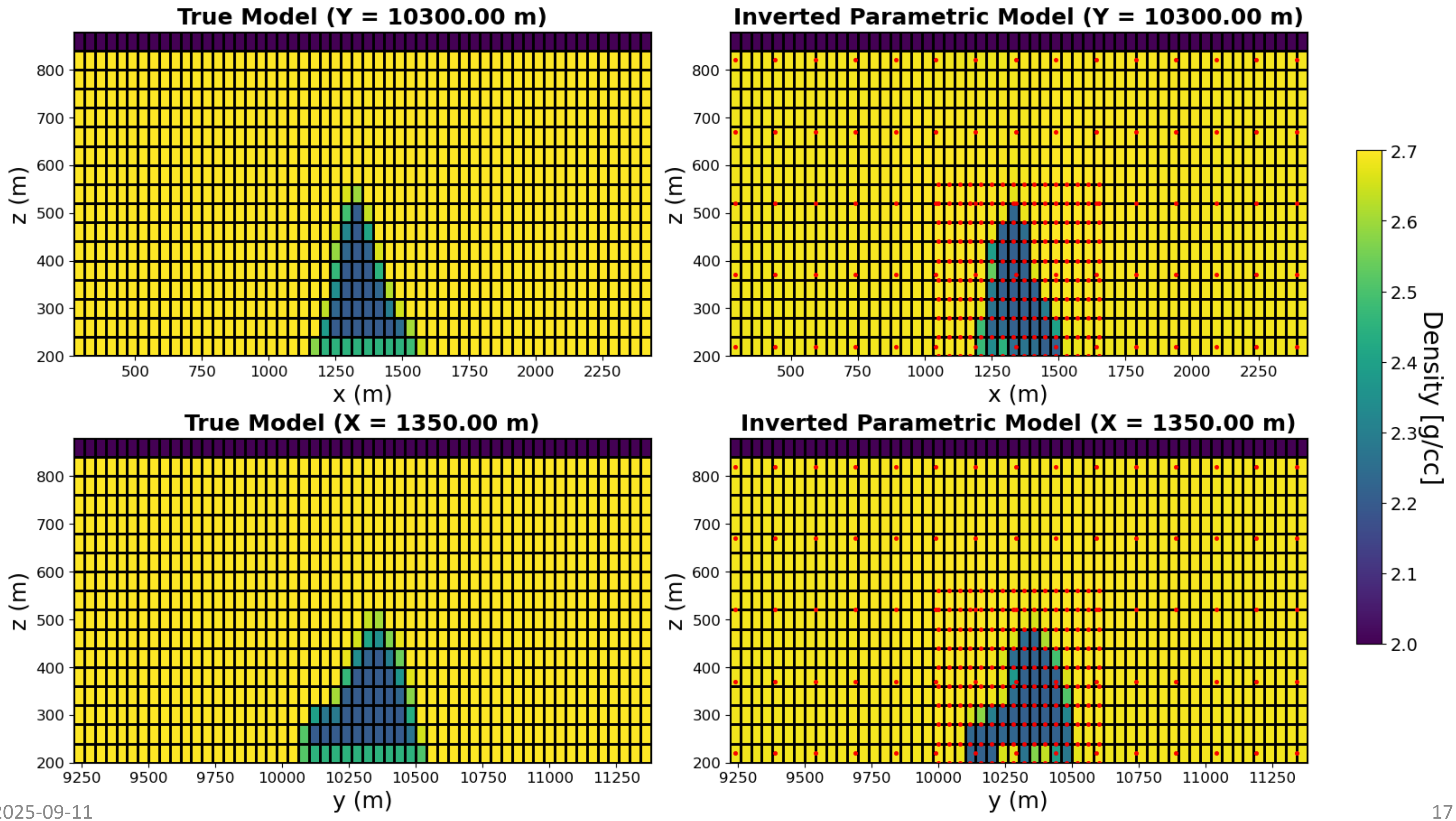
L2 inversion



Recovered param model



Results – Muon Tomography

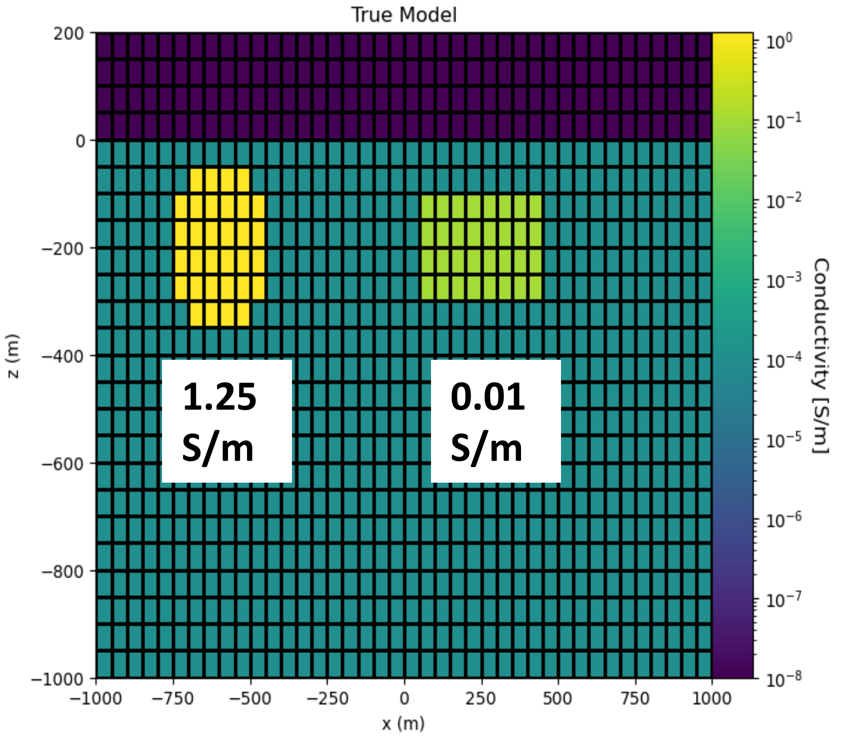


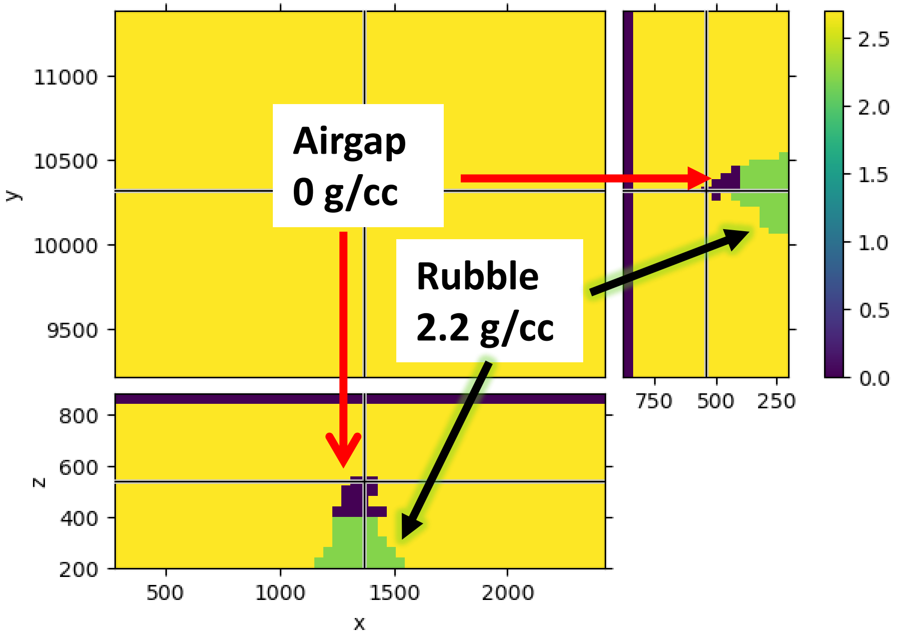
Insights

- Any shape recovery possible without need to specify number of shapes in initial model.
- Less sensitive to starting model vs parametric shapes.
- Method-agnostic: applicable to EM, gravity, magnetics, seismic, etc.
- Flexibility by adding various parameters.
- Can be combined with cooperative inversion to determine background.
- Challenges
  - Requires more total iterations.
  - Multiple bodies with differing physical properties.

Current Focus

- Inverting for bodies with different physical property contrasts.
- Approaches:
  - Add additional level set for each body
  - Add contrasts as parameters

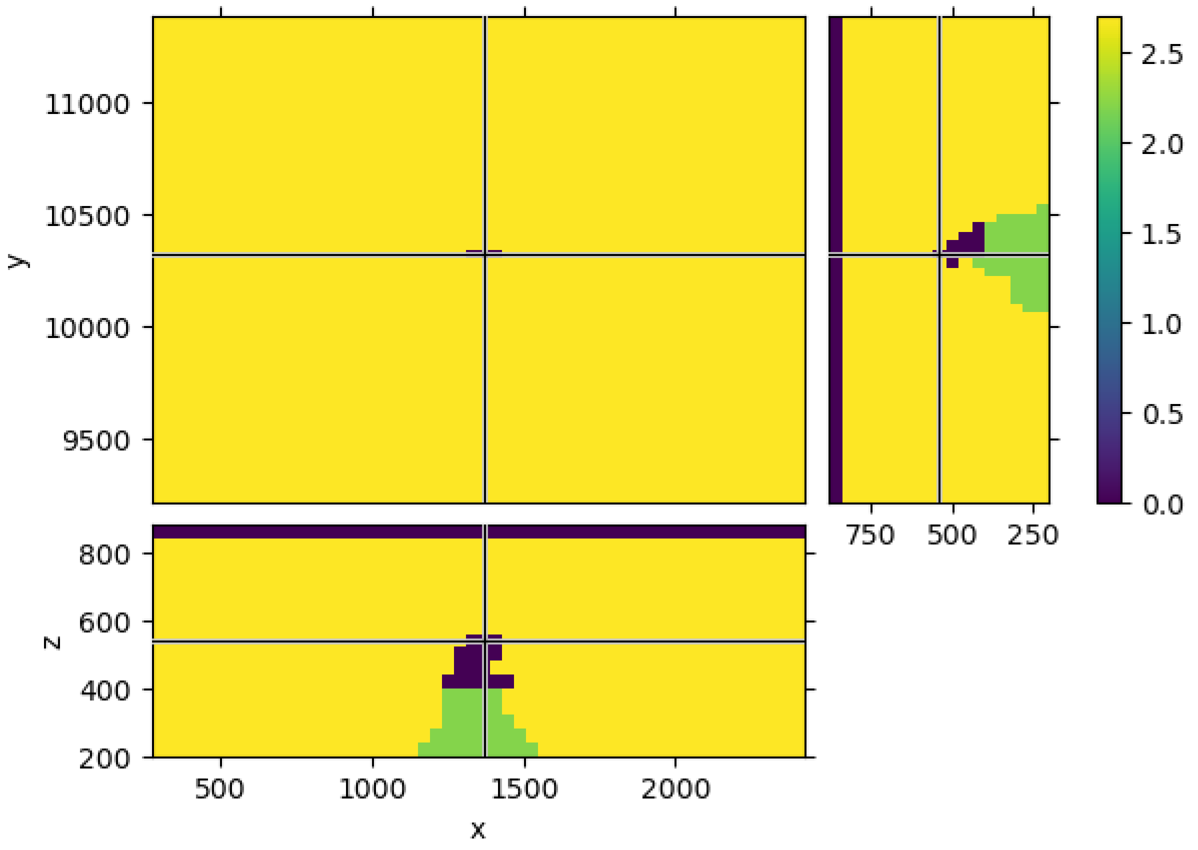




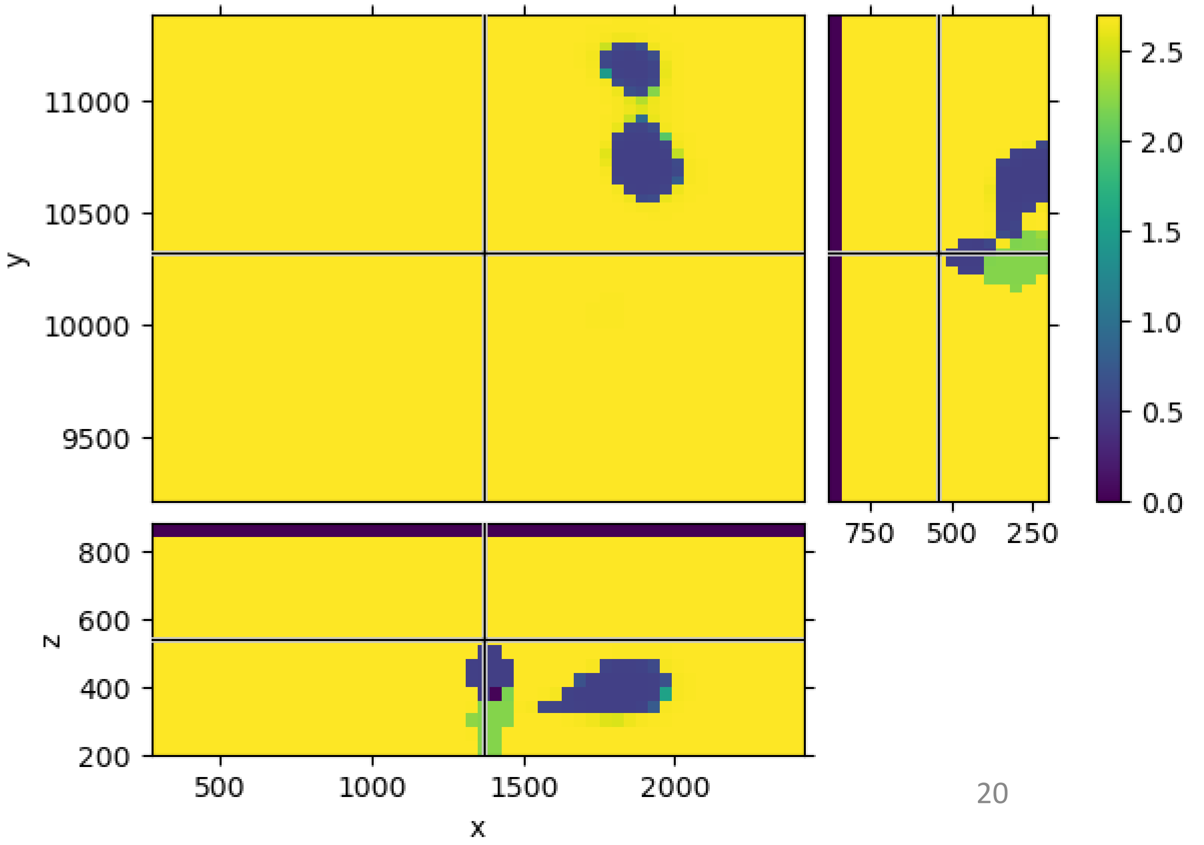
Current Focus – Multicontrast

- Implementing multiple contrasts from two level-set functions.
- Doubles # of unknowns and requires knowing # of bodies.

true model



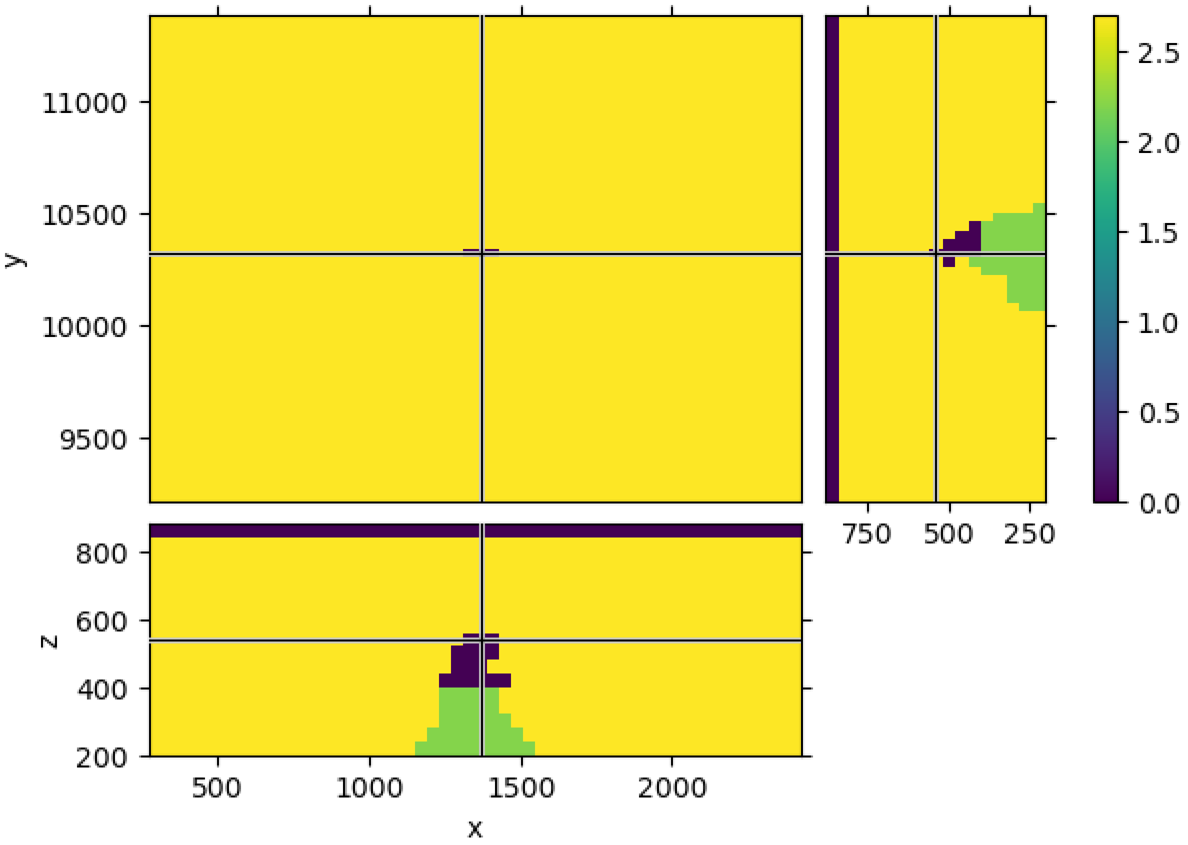
recovered model



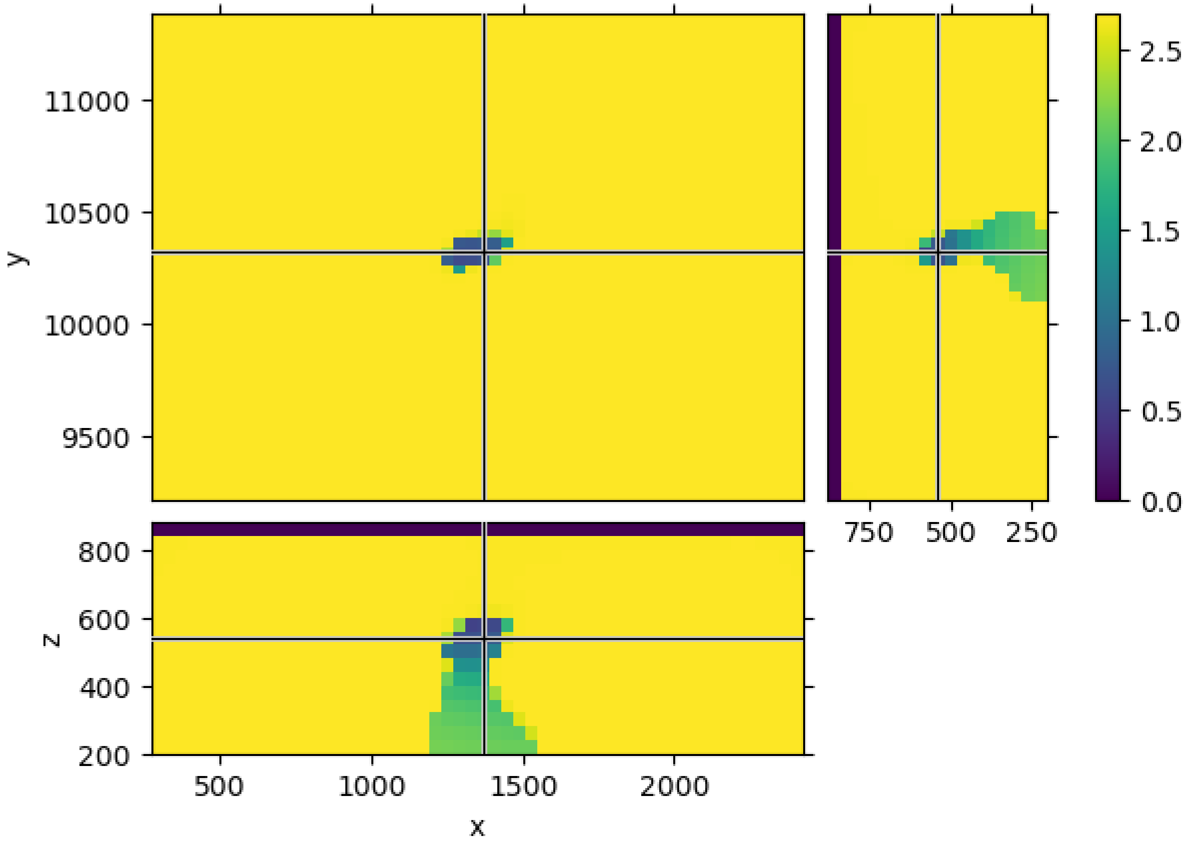
Current Focus – Multicontrast

- Implementing multiple contrasts from one level-set function.

true model



recovered model



Acknowledgements

simpeg

- IMAGE ’25
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- SimPEG community
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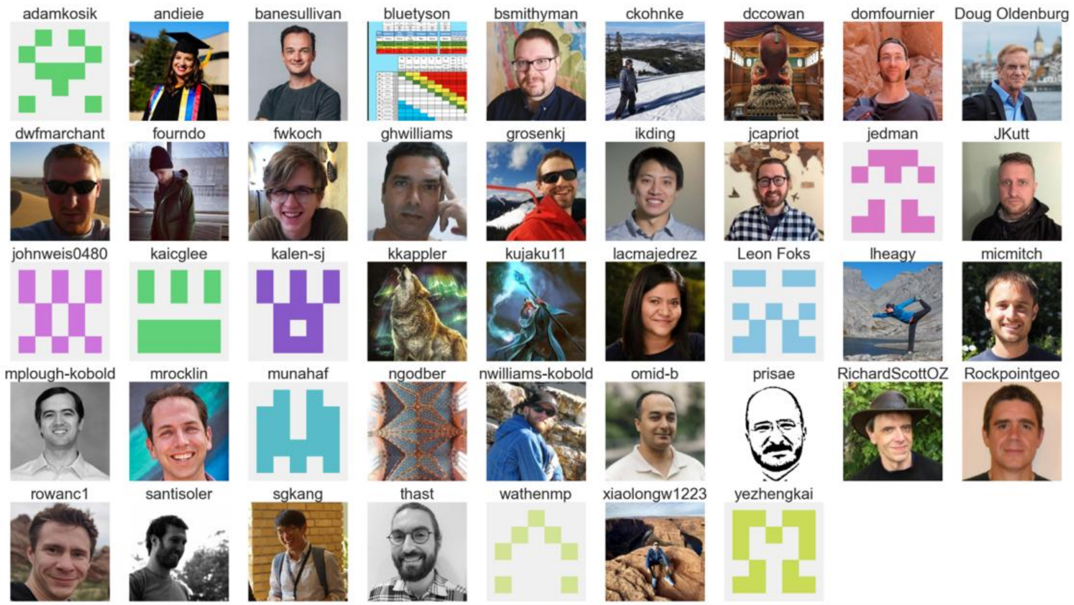

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